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Economic Indices for the North Pacific Groundfish Fisheries: Calculation and Visualization

by
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U.S. DEPARTMENT OF COMMERCE
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Abstract

This report details the methods used to create indices for monitoring economic performance in the Alaskan North Pacific Groundfish Fisheries published in the annual Economic Status of the Groundfish Fisheries off Alaska report. The intuition and interpretation of the indices is discussed followed by a review of the formal literature on the technical properties of indices and the methods for their construction. A decomposition of the Fisher Index is derived which relates sub-indices to a larger aggregate index. The derivations are extended to chained indices over time. A case study of the Gulf of Alaska shoreside groundfish fishery is used to show how the indices and supporting statistics can be graphically displayed to efficiently characterize significant amounts of data across different dimensions of economic markets.

Contents

Introduction

This report describes the methods used to calculate the economic indices presented in the annual Economic Status of the Groundfish fisheries off Alaska report (Economic SAFE) (Hiatt et al. 2011) chapter of the North Pacific Groundfish Stock Assessment and Fisheries Evaluation Report (Groundfish SAFE) (Aydin et al. 2011). Economic indices were first introduced in the 2010 Economic SAFE and will be updated annually to track economic changes in the fisheries.

The Groundfish SAFE contains detailed information on the status and health of the fish stocks, ecosystem, and the economic conditions of the North Pacific fisheries. These data are useful for monitoring and management decisions. The Economic SAFE in particular documents annual economic statistics collected including ex-vessel and first-wholesale value (revenue), production (catch or post-processing production) and prices. Other economic factors such as discard rates and prohibited species catch are also documented in the Economic SAFE. The bulk of the Economic SAFE's content is devoted to reporting data using tables stratified across regions, species, gear-types, product types, and other factors. The indices developed and implemented in this technical report complement these data by giving a comprehensive and interpretable synthesis of the changes in core economic variables in Alaska fisheries.

Indices measure changes over time in the levels of an economic variable (e.g., price) for a set of related goods in a market. The index aggregates the set of variables to provide a single number that is meant to summarize its state in the market. This aggregation is done in a way that achieves two objectives: the first is that classes of goods (e.g., pollock products) that make up a larger proportion of the market should have greater weight in the index; the second is that the index should be comparable over time and across related indices. Indices and the methods used to construct them have foundations in both mathematics and economics. Diewert and Nakamura (1993) gives in-depth treatment of the the various economic index methodologies. The index methods reviewed and developed in this manuscript focus on the Fisher index (Fisher 1922) which are used in the Economic SAFE , and its component indices the Laspeyres Index (Laspeyres 1871) and the Paasche Index (Paasche 1874).

There are many dimensions of a market that one could imagine characterizing through economic indices. Prices indices, could for example, be created for each of the species sold in a market, or each of the product types (e.g., fillets), or even each of the gear types used to harvest a species. Hypothetically, an index can be created for any well-defined class of goods. Alternatively, an "aggregate index" could be created for all goods in a market. The many goods and classes of goods cumulated over in the aggregate index may be extensive. Because sub-indices created from classes which define subsets of the goods cumulated over by the aggregate index, one might naturally consider the relationship between the sub-indices and the aggregate index. We call a set of classes that correspond to a similar characteristic of the good an "attribute". For example, fisheries goods could be classified by the species type, the classes would be the individual species types (e.g., pollock, cod, etc.) and the attribute would be the "species". This report describes a framework for relating the indices of an attribute to the aggregate index through the market share of a class. This relationship has largely been used by practitioners as an aggregation principle for combining multiple indices (e.g., combining regional Consumer Price Indices (CPI) into a national CPI). The market-share relationship also decomposes the aggregate index into multiple sub-indices over classes of the goods for a given attribute.

The next section provides the basic mathematical representation of the indices considered here and provides intuition that is useful for interpreting indices. While quantitative, this section was written for a broad audience. A more formal treatment of indices is presented in the following sections which builds the Fisher index from its component indices and provides a technical economic interpretation of the underlying assumptions and consequent interpretation of an index number. Methods are presented for decomposing aggregate indices into sub-indices along different dimensions of the market and extending the two-period index to the chain index. The index methods developed are applied to the Gulf of Alaska (GOA) shoreside sector as an example.

An Introduction to Economic Indices

An economic index is a summary statistic that normalizes changes over time and provides a unit-free measure of an economic variable. Indices are descriptive, showing a state-of-the-world relative to some base. This section is intended to provide an intuitive understanding useful for interpreting indices. For the readers seeking an understanding of the how the aggregate and sub-indices of an attribute are constructed and connected, the intuition in this section can provide a foundation for the more formal mathematical sections that follow.

The operational interpretation of an economic index is the same for value, price, and quantity indices. An index is a normalized metric that captures changes in some economic series over time. Part of the broad appeal of indices is that little more than this basic understanding is needed to understand a time series plot of an index. For the sake of exposition, we will consider an aggregate price index for the shoreside wholesale market in the GOA but the discussion will apply equally well to the quantity and value indices as well as to the other regions and markets (Table 1).[1] We will write the two period price index between 2009 and 2010 as $P(2010|2009)$. This index gives the aggregate prices in 2010 using 2009 as a reference period. If the price index in 2009 was $P(2009|2008) = 1$ and the price index in 2010 was $P(2010|2009) = 1.1$ then the price index would indicate that when you consider all the prices together for the GOA shoreside wholesale market there was a 10% increase in prices over the year.

There are many species and products that GOA shoreside processors sell on the first wholesale market, including headed-and-gutted sablefish or pollock fillets, which each have their own price. The index $P(2010|2009)$ is formed by taking a weighted sum of the relative prices between 2010 and 2009 over all of these goods: $P(2010|2009) = \sum_{i=1}^{N} \frac{p_i(2010)}{p_i(2009)} * S_i(2010|2009)$. Here, $p_i(2010)$ is the price of good i (e.g., pollock fillets) in 2010 and $S_i(2010|2009)$ is the weight representing the market relevance of good i between 2009 and 2010 in the GOA shoreside wholesale market. The economic measure that is used to determine the market relevance is the proportion of total value that good i makes up in the market, the value share (see

[1] Other species complexes, product groupings, or regions could be used. This paper utilizes the definitions and groupings that are used in the Economic SAFE (Hiatt et al. 2011) for consistency and comparability.

the methods sections for details).

Goods sold in a market have different attributes. For example, goods sold on the wholesale market, product type or species, that represent a way of disaggregating data into meaningful classes are referred to as attributes or attribute dimensions. The goods used to construct the price considered had multiple attributes, classes of these attributes are shared across multiple goods. For example, cod fillets and pollock fillets have the same class “fillet” in the “product type” attribute but differ in the species that they come from. Using the same basic weighting idea we can relate the class indices of an attribute (e.g., species price indices) to the individual components. Consider as an example the price index for a single species, say Pacific cod: $P^{cod}(2010|2009)$. Similar to the aggregate price index we would build the cod price index as a weighted sum of all the cod-based products.

The sub-indices of an attribute can be combined to form an aggregate index. Similar to the construction of the aggregate index, the sub-indices can be combined as a weighted sum where the value share of a class in a attribute serves as the weight:

$$P(2010|2009) = \sum_{j=1}^{J} P_j(2010|2009) * S_j(2010|2009),$$

where j is runs over the species attribute $j \in \{cod, pollock, sablefish, ...\}$ (Table 1) and $S_j(2010|2009)$ can be thought of as an market-relevance weight determined by the value share for the species j proportion of total value. This decomposition of the aggregate index into sub-indices is referred to here as the value-share decomposition. This decomposition can be done for other attributes as well, for example the aggregate price index can be expressed as a weighted sum of the individual product price indices:

$$P(2010|2009) = \sum_{k=1}^{K} P_k(2010|2009) * S_k(2010|2009),$$

where k runs over product types, $k \in \{fillet, head\&gut, surimi, ...\}$ and $S_k(2010|2009)$ is the value share of product k.[2][3]

Comparison of indices across multiple periods is done by chaining consecutive two-period estimates together to create a chain index. Many indices typically encountered in the news are chain indices. Chain indices specify a base period in which the index is equal to 100. Using the GOA shoreside price index as an example, the 2008 chained price index is given by $I(2008|2006) = 100 * P(2007|2006) * P(2008|2007)$. Multiplying this by the two-period price increment between 2008 and 2009, $I(2009|2006) = I(2008|2006) * P(2009|2008)$, chains the index forward. The first year, 2006, is the base year for this chain index.[4] To provide a concrete numerical example, take the 2006 base year index to be equal to 100 and say there was a 50% increase in aggregate prices, $P(2007|2006) = 1.5$. The chained price index in 2007 would be $I(2007|2006) = I(2006|2006)*P(2007|2006) = 100*1.5 = 150$. Now say that there was a 50% decrease in aggregate prices between 2007 and 2008, $P(2008|2007) = 0.50$. The 2007 chained price index would now be $I(2008|2006) = I(2007|2006) * P(2008|2007) = 150 * 0.5 = 75$. Thus, the value of the index in 2008 makes sense with respect to both 2006 and 2007. That is, 2008 prices are 75% of their 2006 level and half their 2007 level. Notice from the price index equation that the weights in the chain index $S_k(t|t-1)$ change over time and hence adapt to to potential shifts in the value share induced by swings in output/production. This is an important feature of the index in fisheries where output can change significantly based on changes in the stock and the Total Allowable Catch (TAC). [5]

As a final note on index interpretation: indices are descriptive, they characterize the state of the markets. They should be used to determine the state of a fishery relative to other periods, display the co-movement of different species, product types, or gear types and relate the relative influence of movements in these attributes to aggregate performance. The indices

[2]The aggregate index constructed by cumulating over sub-indices is equivalent to the index constructed by cumulating over individual goods: $P(2010|2009) = \sum_{i=1}^{N} \frac{p_i(2010)}{p_i(2009)} * S_i(2010|2009) = \sum_{j=1}^{J} P_j(2010|2009) * S_j(2010|2009)$. (see the methods section)

[3]Sub-indices of an attribute for value and quantity ($V_i(t|t-1)$, $Q_i(t|t-1)$) can be similarly constructed and combined as a weighted average over attribute indices.

[4]For chain indices covering longer time spans the base year can be shifted.

[5]The adaptability of weights is a feature of the chain index. This feature is in contrast to fixed-base indices that use a constant weight over time. (see the section describing chained indices)

have no inherent casual interpretation. For example, it would be wrong to assert from these indices change in surimi prices "caused" a change in pollock price. Nor could we say the converse. We can say that they are connected as surimi is a significant portion of the value from pollock in some regions, but causality is beyond the scope of indices. Carefully designed regression analysis is better suited to trying to answer causality questions.

As a matter of terminology production, be it from processors or from catcher-vessels, will be generically referred to as the quantity produced, or simply the "quantity". Thus, a quantity index could also be called a production index. Similarly, value could more accurately be referred to as revenue (e.g., ex-vessel revenue or first wholesale revenue). Again, the conventional nomenclature and notation of using the term value and the variable V will be maintained here. It bears mentioning that product revenue does not constitute the only source of value from a fishery. There are many other direct and indirect source of value (both negative and positive) that may come from catching/producing fish. Additionally, revenues or "value" do not incorporate costs, and as such the value index should not be interpreted as a profit index.

Methods for the Construction of Economic Indices and Sub-indices

Indices have a long history in both economics and mathematics (Diewert 1988). The development of indices in these two fields has resulted in two approaches to the theoretical interpretation and construction of indices. The mathematical or test approach builds indices that satisfy a number of desirable properties. The economic approach, also called the functional approach, constructs the index as an object that is representative of an economic variable from utility/profit maximizing behavior of the underlying agents. The two approaches are not mutually exclusive, and the test framework is useful for thinking about the mechanical properties of the economic indices. Thus, while the focus remains on economic indices the test framework are utilized where necessary provide a better understanding.

The consideration of value serves as a starting point for the creation of economic indices. For any good its direct market value can be expressed as the quantity of a good sold at a given price $v = p \times q$. The ratio of values between periods (1 and 0), $V = V(1|0) =$

$v(1)/v(0)$, is a unit-free index of the change in value. The power of indices is their ability to aggregate information over multiple goods that belong to a similar class. Any reasonable definition can be used to specify a class and restrict the scope of the index to a desired subset of goods. The value index then becomes the ratio of relative revenues (value) between the two periods for the desired class of goods (Coelli et al. 2005):

$$V(1|0) = \frac{R(p(1), q(1))}{R(p(0), q(0))} = \frac{\sum_i v_i(1)}{\sum_i v_i(0)}, \tag{1}$$

where $R(\cdot)$ is the revenue function that takes time t prices $p(t)$ and quantities $q(t)$ as its arguments and i subscripts define classes of goods.

The prices and quantities that make up the values each period are the endogenous realization of market activity for the goods (e.g., species, product types) being indexed. The observed prices and quantities were (to varying degrees depending on the market) simultaneously determined by agents (e.g., harvesters and processors or processors and wholesale purchasers) in the market place. A multitude of other factors such as production technologies, prices of production inputs, and global demand, which is collectively called the state-of-the-world, influenced the decisions that led to the observed prices, quantities, and consequent value. The perfect (infeasible) price index $P^*(1|0)$ would hold constant the state-of-the-world changing only the price between periods 0 and 1:

$$P^*(1|0) = \frac{R^*(p(1), q')}{R^*(p(0), q'')}, \tag{2}$$

where the $*$ indicates the constant state-of-the-world and q' and q'' are the revenue maximizing quantity under $*$ for market participants facing prices $p(1)$ and $p(0)$.[6] Thus, in the hypothetical perfect index, different prices allow for optimizing market participants to change production and purchasing decisions. While infeasible, it is useful for both formalizing what it means to be an economic index and for considering the relative merit of competing indices. Similarly, the perfect quantity index $Q^*(1|0)$ is the ratio of revenues for differing quantities holding the state-of-the-world constant.

[6] Because it's assumed that producers maximize revenue the observed value is the maximized value when prices and quantities from the same period are used: $R^*(p(t), q(t)) = R(p(t), q(t))$.

The perfect price index is infeasible because the observed quantities in periods 0 and 1 are the realized quantities determined in each period's respective state-of-the-world. The construction of a feasible economic index has lead to the formulation of many alternative indices such as Laspeyres' (Laspeyres 1871) and Paasche's (Paasche 1874) indices, the Fisher Index (Fisher 1922), the Törnqvist Index (Törnqvist 1976), or the Malmquist indices (Malmquist 1953), each having their own merit. Many address particular economic scenarios by modeling economic decisions which can in turn imply a particular index. For example, by assuming a trans-log production function model one can show that the Törnqvist Index is exact. The Fisher Index can be derived as the exact index for a quadratic production technology. Both the Törnqvist and Fisher indices give a second-order approximation to the ideal economic indices derived directly from any standard (smooth) production functions (discussed further in the subsection describing the Fisher Index).

I follow the standard approach to presenting of basic index theory and begin with the Laspeyres' and Paasche's indices. They are slightly less flexible from a theoretic standpoint, but their simple calculation and interpretation due to their linearity, as well as their practical flexibility has led to their continued use. Together these two indices are used to motivate and construct the popular Ideal Fisher Index which is the type of index used for the North Pacific Groundfish indices.

The Laspeyres and Paasche Indices

In addition to period 0 and 1 the price index (Equation (2)) requires quantities. A natural practical candidate is the base period quantity $q(0)$ which results in Laspeyres' Index:

$$P^L(1|0) = \frac{R^0(p(1), q(0))}{R^0(p(0), q(0))} = \frac{\sum_i p_i(1)q_i(0)}{\sum_i p_i(0)q_i(0)}. \tag{3}$$

Laspeyres' Index implicitly uses the period 0 state-of-the-world, indicated by the superscript $R^0(\cdot)$. Equation (3) shows that the choice of quantity can in some sense be viewed as presenting a counterfactual. From this perspective, Laspeyres' Index captures the change in relative revenue that can be attributed to a change in prices using base period quantities as the counterfactual. It attempts to answer to the question: "what would revenues had been in the base period if prices were the same as prices are today?" As previously discussed, it falls short of answering this question exactly because $q(0)$ was determined in the period 0 state-of-the-world by market participants facing prices $p(0)$. They may have chosen different quantities facing prices $p(1)$, furthermore, the quantities chosen should lead to higher revenues if agents are optimizing. Thus, comparing Laspeyres' Index (Equation (3)) to the prefect price index (Equation (2)) $R(p(1), q(0)) \leq R(p(1), q')$, hence Laspeyres' Index is no bigger than the true price index $P^L(1|0) \leq P^*(1|0)$.

Laspeyres' Index enjoys two properties critical to a useful price index. First, it can capture inflation, a general rise in prices. This can be verified by noting that if $p_i(1) = \alpha p_i(0) \forall i$ (i.e., all prices only change by inflation), where $\alpha > 0$ is the inflation factor, then $P^L(1|0) = \alpha$. Second, the index is independent of the unit of measurement for either prices or quantities as the units cancel in both the numerator and the denominator.

Paasche's price index provides the temporally diametric perspective for the two-period index. It uses the other natural practical candidate available, the current period quantities, $q(1)$, as the revenue counterfactual:

$$P^A(1|0) = \frac{R^1(p(1), q(1))}{R^1(p(0), q(1))} = \frac{\sum_i p_i(1) q_i(1)}{\sum_i p_i(0) q_i(1)}. \tag{4}$$

Analogous to above, Paasche's Index implicitly uses the period 1 state-of-the-world, indicated by the superscript $R^1(\cdot)$. Mechanically, it compares today's (period 1) realized revenues to what revenues would have been earlier (period 0) had the goods being indexed sold at yesterday's prices. Another interpretation is that it attempts to answer the question: "How much of today's revenues can be attributed to prices changes from yesterday's prices?" Paasche's Index falls similarly short of this goal because it fails to account for the quantity adjustment in the market when prices change. Also like Laspeyres' Index, Paasche's Index (Equation (4)) serves as a bound for the perfect price index (Equation (2)), as $R(p(1), q(0)) \leq R(p(1), q')$ implies that Paasche's Index is no less than the true price index $P^A(1|0) \geq P^*(1|0) \geq P^L(1|0)$.

Just as value of a specific good is the product of the prices and quantities, economic indices should satisfy this same basic relationship, $V = P \times Q$, where Q is the quantity index,

P is the price index and V is the value index. Among the properties that this ensures is that marginal changes in the value index are simply the marginal price index change times the quantity index, $\Delta V = (\Delta P) \times Q$. Thus, after defining the price index as either the Laspeyres or Paasche index the corresponding quantity index can be derived implicitly from definition of the relationship between the value, price and quantity indices, $Q = V/P$. The quantity index derived as such sometimes referred as the dual quantity index for the value problem (Coelli et al. 2005). It can be shown algebraically that the dual quantity index implied by using Laspeyres' price index (Equation (3)) is Paasche's quantity index. Similarly, the dual quantity index for the Paasche's price (Equation (4)) is Laspeyres' quantity index:

$$Q^A(1|0) = \frac{V(1|0)}{P^L(1|0)} = \frac{R(q(1), p(1))}{R(q(0), p(1))} \quad Q^L(1|0) = \frac{V(1|0)}{P^A(1|0)} = \frac{R(q(1), p(0))}{R(q(0), p(0))}. \tag{5}$$

From a test perspective both Laspeyres' and Paasche's indices fail what is referred to as the factor-reversal test. That is, the form of the dual implicit index for the value problem does not take the form of the original index. The different temporal counterfactual embedded Laspeyres' and Paasche's indices makes the interpretation of the product of price and quantity indices unclear. The lack of symmetry from the factor reversal test (discussed below further: Test 7) lead Fisher to propose what subsequently became known as the Fisher's Ideal index (Fisher 1922), or simply the Fisher Index. The Fisher Index is constructed by taking the geometric average of Laspeyres' and Paasche's indices. This operation produces an index that satisfies the factor-reversal test. Furthermore, since the Fisher Index is the average of indices that bound the perfect index (Equation (2)) it provides a higher order approximation (i.e., it is closer) to the theoretical true economic index. Fisher's Index has many desirable properties from both an economic and an axiomatic standpoint (Diewert and Nakamura 1993). Before formally presenting the Fisher Index the index properties, which have been touched on already are presented in the next section.

The Properties of an Index

Economic indices can be formalized as a set of properties that an index could satisfy. Not all indices satisfy every property, nor should they; some properties may be less important or irrelevant to the application of an index in a given context. The strengths and weaknesses of the many different indices available are often related to the satisfactions of some subset of these properties. Testing which properties an index satisfies is a way of determining the relative utility of different index formulas. (Diewert and Nakamura 1993, Chapter 2) documents the history of this "test approach" to judging indices.

The following criterion serve as the basis for the test approach to index construction:

Criterion. Tests for economic indices (based on Coelli et al. (2005))

Let $I(t|s)$ be a period-t index with base period s, which indexes variable x and potentially uses variable y to construct weights ($I(t|s)$ could represent either a price, quantity and value index). Variables p,q and v represent price, quantity and value and P, Q and V their respective indices.

Test 1- *Positivity:* $I(t|s) > 0$.

Test 2- *Continuity:* $I(t|s)$ is continuous in prices and quantities.

Test 3- *Proportionality:* If all current prices (quantities) change by α then the index also changes by α, $\tilde{p}(1) = \alpha p(1) \implies I(\tilde{t}|s) = \alpha I(t|s)$. ($p(0)$ is fixed).

Test 4- *Commensurability:* The index is independent of the units of measurement used for prices and quantities. If $\tilde{x} = \alpha x$ is a change of units for $t = 0, 1$ the index constructed with $\tilde{x}$ is unchanged, $\tilde{I}(t|s) = I(t|s)$.

Test 5- *Time Reversal:* $I(1|0)$ = $(I(0|1))^{-1}$.

Test 6- *Mean-value Test:* Let $I^{min} = min({}^{x_i(1)}/_{x_i(0)})$ and $I^{max} = max({}^{x_i(1)}/_{x_i(0)})$ then $I^{min} \leq I(t|s) \leq I^{max}$.

Test 7- *Factor-Reversal Test:* Let $f(x(t), x(s), y(t), y(s))$ be an index formula. The factor reversal test is satisfied if $P(t|s) = f(p(t), p(s), q(t), q(s))$, $Q(t|s) = f(q(t), q(s), p(t), p(s))$ and $P(t|s) * Q(t|s) = V(t|s)$ where $V(t|s) = {}^{v(t)}/_{v(s)}$.

Test 8- *Transitivity Test:* For time periods t,s, and r an index is transitive if $I(t|r) = I(t|s) * I(s|r)$.

Tests 1-4 and 6 are basic properties for an index. Tests 1 and 2 are positivity and smoothness constraints. The proportionality Test 3 examines whether the index captures inflation (an overall increase in prices) and hence can be inflation adjusted by conventional means. Commensurability (Test 4) ensures that the index is unit-free. The mean-value test (Test 6) examines whether the index is bounded by the minimum and maximum ratio of the variable being indexed.

The relevance of Tests 5, 7 and 8 depends on the context in which the indices are to be used and tend to be the tests that indices may fail. The time-reversal test (Test 5) is a temporal consistency test which is not satisfied by either Laspeyres' or Paasche's indices. The factor-reversal test (Test 7) proposed by Fisher (Fisher 1922) ensures the methodological consistency when simultaneously constructing price, quantity and value indices. The factor-reversal test ensures that the price and quantity indices were constructed under the same temporal counterfactual discussed previously. Indices satisfy the transitivity property (Test 8) (also known as the circularity test) if the index takes on the same value in separate time periods when prices and quantities are the same. Few indices satisfy the transitivity test without some adjustment (known as reflection). This property has been invoked most often for spatial indices where the relation of one spatial unit to another through a third is necessary. [7]

The Fisher Index

Fisher's Ideal Index (Fisher 1922) is obtained by taking the simple (unweighted) geometric mean of Laspeyres' (Equation (3)) and Paasche's (Equation (4)) indices for price and quantities, respectively:

$$P^F(1|0) = \sqrt{P^L(1|0) \times P^A(1|0)} \quad Q^F(1|0) = \sqrt{Q^L(1|0) \times Q^A(1|0)}. \tag{6}$$

The Fisher Index satisfies Tests 1-7, failing only the transitivity test (Test 8). The Fisher Index doesn't have a single-period counterfactual as with Laspeyres' and Paasche's indices. The intuitive appeal derives more from the shortcomings of Laspeyres' and Paasche's indices in that it is a central measure of two valid indices which bound the"true" economic index. Economic theory aside, in practice, Laspeyres' Index can be numerically greater

[7]See Diewert and Nakamura (1993) for further discussion of index tests and Balk (1995) for price indices in particular.

than or less than Paasche's Index. This largely depends on how the index weighting variable (e.g., quantity for the price index) are changing between the two periods. Note that when quantities between period 0 and 1 are the same the Laspeyres and Paasche price indices, and hence the Fisher price index, are identical. Furthermore, regardless of whether or not Laspeyres Index is larger than the Paasche Index, numerically they bound the Fisher Index. This can be easily seem from the Fisher Index being defined as the geometric mean, $min(I^L, I^A) \leq I^F = \sqrt{I^L * I^A} \leq max(I^L, I^A)$.[8]

Another strength of the Fisher Index as an economic index is that it is, what Diewert (1976) calls, superlative. It is capable of providing a second order differential approximation to an arbitrary twice-differentiable linearly-homogeneous aggregator function (e.g., a production function). Thus, no functional form for the production function need be assumed in order for the Fisher Index to be valid, only some smoothness and consistency properties. Additionally, if the "homogeneous quadratic aggregator", a flexible functional form, is used for the production function, then the Fisher Index is the "exact" theoretic index for capturing changes in prices/quantities.

The economic indices for North Pacific groundfish present values, quantity, and price indices together. Thus, an index method which satisfies the factor-reversal test is needed. Furthermore, we would also highly value its ability to closely track the "true" economic index and its flexibility in representing the productivity of a variety of underlying fishing sectors. Because the Fisher Index has these properties and it is used to construct the economic indices for the North Pacific groundfish fisheries.

Relating Disaggregate and Aggregate Economic Indices

This section derives the value-share decompositions for the Laspeyres, Paasche, and Fisher indices. This decomposition technique is a reinterpretation of a common method for aggregating indices that uses expenditure (or value-share) weights to aggregate multiple indices over consumers or producers. The decomposition is carried out over dimensions of the

[8] Additionally, recall that the relationship between the arithmetic and geometric mean. $\frac{I^L + I^A}{2} \geq \sqrt{I^L * I^A}$ with equality when $I^L = I^A$.

market corresponding to attributes of the good considered. For this reason, I refer to decomposition derived in this section as a "attribute decomposition". Alternatively, the creation of the index could be looked at from the perspective of attribute aggregation. The attribute decomposition is similar to the property of consistency in aggregation (Diewert and Nakamura 1993, Chapter 9).

Indices that are consistent in aggregation can be calculated in two stages by first calculating sub-indices over entities (say, regionally, as might be done with the CPI) then calculating an aggregate index over the sub-indices, or equivalently in one stage by calculating the aggregate index over all the individual agent/goods.

Because the decomposition applies equally to the value, price and quantity indices the following notation will be adopted. Let I be a value, price or quantity two-period index, I^L be the Laspeyres Index, I^A be the Paasche Index, and $I^F = \sqrt{I^L * I^A}$ be the Fisher Index. Time periods $t = 0$ and $t = 1$ are the "base" and "current" periods, respectively. Because there are only two periods the notation indicating an index's functional dependence on time is condensed ($I(1|0) = I$) to avoid clutter. The index will be constructed for the variable $x(\cdot)$ and weights may be calculated using the additional variable $y(\cdot)$.

A precise definition of an attribute and class for a good will lay the foundation for the attribute decomposition. For simplicity the definition uses goods that have two attributes, however, the definition extends in a straightforward way to any number of classes.

Definition. Attribute and class of a good

> Let $x_{j,k}$ be a set of goods enumerated by $j \in J$ and $k \in K$. Define the sets J and K as attribute dimensions. A class is a given $\tilde{j} \in J$ and the set of goods in the class $\tilde{j}$ are $\{x_{j,k} \ s.t. \ j = \tilde{j}\}$. The classes of attribute K are defined in the same way.

When there are two attributes, J and K the variables $x_{j,k}$ can be thought of as elements of a two-dimensional matrix where non-existent goods are zero valued elements in the matrix. A class is then all of the goods in a single row or column vector. As an example, consider prices for fisheries goods with attributes being species and product type. The set of prices in the class Pacific cod are the prices across different product types for the species is cod. Similarly, the fillet class is constructed from the prices across species for the product type fillet.

Individual indices over the classes of an attribute can be independently created along the multiple attribute dimensions. The independent[9] class indices can be related to the aggregate index, which uses all the data from the individual classes, establishing a link to changes in the markets for the classes and broader scale changes at the aggregate. The attribute decomposition shows the functional relationship between the individual class indices and the aggregate index. The two-period Laspeyres Index has an exact value-share decomposition.

Proposition 1. Value-share decomposition of the Laspeyres Index

> Let J and K be attributes as defined above and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Laspeyres Index $I^L(1|0)$ over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^L(1|0)$ over an attribute J or $I_k^L(1|0)$ over attribute K.

$$\begin{aligned}
&I^L(1|0) = \frac{\sum\limits_{j\in J}\sum\limits_{k\in K} R_{j,k}(x_{j,k}(1),y_{j,k}(0))}{\sum\limits_{j\in J}\sum\limits_{k\in K} R_{j,k}(x_{j,k}(0),y_{j,k}(0))} = \sum_{j\in J} I_j^L(1|0)*S_j^L = \sum_{k\in K} I_k^L(1|0)*S_k^L \\
&\text{where} \quad I_j^L(1|0) = \frac{R_j(x_{j,k}(1),y_{j,k}(0))}{R_j(x_{j,k}(0),y_{j,k}(0))} \quad S_j^L = \frac{R_j(x_{j,k}(0),y_{j,k}(0))}{\sum\limits_{j\in J} R_j(x_{j,k}(0),y_{j,k}(0))} \\
&\text{and} \quad R_j(x_{j,k}(t), y_{j,k}(s)) = \sum_{k\in K} R_{j,k}(x_{j,k}(t), y_{j,k}(s)) \\
&\text{with} \quad I_k^L,\ S_k^L,\ R_k(x_{j,k}(t), y_{j,k}(s)) \quad \text{equivalently defined.}
\end{aligned} \tag{7}$$

Proof: See Appendix.

Thus, the sum of the independent Laspeyres sub-indices $I_j^L = \frac{R_j(x_{j,k}(1),y_{j,k}(0))}{R_i(x_{j,k}(0),y_{j,k}(0))}$ (Equation (3)) weighted by the base-period value shares for class j $S_j^L = \frac{R_j(x_{j,k}(0),y_{j,k}(0))}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(0))}$ yields the aggregate index. Alternatively, the aggregate index can be decomposed as a linear combination of independent class indices for a given attribute where the value-share are the coefficients on the class indices.

Paasche's Index has a similar linear decomposition. Like the difference between the simple indices, Laspeyres and Paasche decomposition differ in the value-share weight relating the sub-indices to the aggregate index.

---

[9] The word 'independent' is used here to indicate that one index can be constructed without constructing the others and the data used in one class index is mutually exclusive of the data used in another class index within a given attribute. Statistical or causal independence is not assumed or implied here.

Proposition 2. Value-share decomposition of the Paasche Index

Let J and K be attributes as defined above and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Paasche Index $I^A(1|0)$ over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^A(1|0)$ over an attribute J or $I_k^A(1|0)$ over attribute K.

$$\begin{aligned}
&I^A(1|0) = \frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1),y_{j,k}(1))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0),y_{j,k}(1))} = \sum_{j\in J} I_j^A(1|0)*S_j^A = \sum_{k\in K} I_k^A(1|0)*S_k^A \\
&\text{where} \quad I_j^A(1|0) = \frac{R_j(x_{j,k}(1),y_{j,k}(1))}{R_j(x_{j,k}(0),y_{j,k}(1))} \quad S_j^A = \frac{R_j(x_{j,k}(0),y_{j,k}(1))}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(1))} \\
&\text{and} \quad R_j(x_{j,k}(t), y_{j,k}(s)) = \sum_{k\in K} R_{j,k}(x_{j,k}(t), y_{j,k}(s)) \\
&\text{with} \quad I_k^A,\ S_k^A,\ R_k(x_{j,k}(t), y_{j,k}(s)) \quad \text{equivalently defined.}
\end{aligned} \tag{8}$$

Proof: See Appendix.

This decomposition can be viewed as relating the independent Paasche sub-indices over classes for a given attribute to the aggregate Paasche Index or as decomposing the aggregate index into a linear combination of class indices for a given attribute with S_j^A as the coefficient on the class indices. The value share S_j^A in the Paasche Index is neither a current nor base-period value share. An algebraic manipulation of the Paasche's Index shows that it can be thought of as using current-period weights if we consider the index as a weighted harmonic mean. The harmonic mean representation of the simple Paasche Index (Equation (4)) is well-known(Coelli et al. 2005) and it holds for the attribute decomposition as well:

$$\begin{aligned}
I^A(1|0) &= \left(\frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0),y_{j,k}(1))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1),y_{j,k}(1))}\right)^{-1} = \left(\sum_{j\in J} \frac{R_j(x_{j,k}(0),y_{j,k}(1))}{R_j(x_{j,k}(1),y_{j,k}(1))} * \frac{R_j(x_{j,k}(1),y_{j,k}(1))}{\sum_{j\in J} R_j(x_{j,k}(1),y_{j,k}(1))}\right)^{-1} \\
&= \frac{1}{\sum_{j\in J} \hat{S_j^A}/I_j^A(1|0)} \quad \text{where} \quad \hat{S_j^A} = \frac{R_j(x_{j,k}(1), y_{j,k}(1))}{\sum_{j\in J} R_j(x_{j,k}(1), y_{j,k}(1))}.
\end{aligned} \tag{9}$$

The weighted harmonic mean decomposition is analogously a weighted central measure of the Paasche attribute indices, $I_j^A(1|0)$. While the specific form of the value-shares for harmonic mean and simple linear decompositions differ slightly, both the aggregate index from both is identical: $\sum_{j\in J} I_j^A(1|0)*S_j^A = I^A(1|0) = \frac{1}{\sum_{j\in J} \hat{S_j^A}/I_j^A(1|0)}$. For the decomposition of the Fisher Index the linear Paasche decomposition is a more useful representation.

The decompositions of the Laspyres and Paasche indices above are derived using variables with only two attribute dimensions. These derivations extend to goods with more than two attribute dimensions because the aggregation of indices is a commutative composition of functions. In the two attribute case $f_J(f_K(I_{j,k})) = I = f_K(f_J(I_{j,k}))$, where $f(\cdot)$ is the index aggregator function. Iterative application of this property extends the decompositions to the case where there are more than two attributes. The equivalence between decompositions is mechanically just a result of the fact that the order in which things are added is irrelevant, they still sum up to the same thing.

In contrast to the Laspeyres and Paasche indices, which relate the attribute indices to the aggregate through a linear function, the Fisher Index is a nonlinear function. To achieve the linear attribute decomposition of the Fisher Index, a first-order approximation is used. The approximation is made about the base period making it an approximation of the Fisher Index increment. Diewert and Nakamura (1993) shows that a first-order approximation of the Fisher Index has a small margin of error and is actually second-order exact. Furthermore, a first-order approximation yields an intuitive interpretation of the index as the weighted sum of the percentage change in prices. Because the Fisher Index is the geometric mean of Laspeyres' and Paasche's indices, the weights relating the Fisher sub-indices to aggregate index are an average of the Laspeyres' and Paasche's weights. While other papers have made first-order approximations to the Fisher Index, the use of the first-order approximation to relate the sub-indices to the aggregate index is the first that I am aware of and provides new insight into the Fisher Index.

Proposition 3. Value-share decomposition of the Fisher Index

> Let J and K be attributes as defined above and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Fisher Index $I^F(1|0)$ over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^F(1|0)$ over an attribute J or $I_k^F(1|0)$ over attribute K.

$$
\begin{aligned}
&I^F(1|0) = \sqrt{I^A(1|0) * I^L(1|0)} = \sum_{j \in J} I_j^F(1|0) * S_j^F = \sum_{k \in K} I_k^F(1|0) * S_k^F \\
&\text{where} \quad I_j^F(1|0) = \sqrt{I_j^A(1|0) * I_j^L(1|0)} \quad S_j^F = \tfrac{S_j^A + S_j^L}{2} \\
&\text{with } I_j^L(1|0), S_j^L \text{ defined as in Equation (1) and} \\
&I_j^A(1|0), S_j^A \text{ defined as in Equation (2)} \\
&\text{The } K \text{ attribute decomposition using } I_k^F,\ S_k^F \text{ is equivalently defined.}
\end{aligned} \tag{10}
$$

Proof: See Appendix.

The attribute decomposition of the Fisher Index shows that it is value-share weighted average of the percentage changes in each of the index variables where the share weight is the average of the Laspeyres and Paasche Index value shares. We can make the attribute decomposition look more like the Laspeyres and Paasche decompositions by defining $S_i^F \equiv \frac{1}{2}\left(S_i^L + S_i^A\right)$.

Chaining Indices and Sub-Indices Over Time

Two-period indices provide a foundation for constructing indices. Often, and in the present context, the complete time series of an index is needed so that movements in earlier periods are comparable to movements in later periods. For the economic indices the North Pacific Groundfish Fisheries the method of chaining indices is used. This section shows how a chain index is calculated and explains how different time periods can be compared and interpreted. The relationship between the aggregate chain index and sub-indices for an attribute is derived followed by a discussion of the differences between chained and fixed-base indices, the alternative method for calculating indices.

Chaining integrates the increments of the index over the entire period to calculate a time series (Diewert 2011, Chapter 8). This is accomplished by iteratively multiplying consecutive two-period indices together as a chain, where each link in the chain is a two-period index. Thus, for the sequence of two-period indices $I(1|0), I(2|1), I(3|2), \ldots, I(T|t–1)$ the chained index at each period $t \in 0, 1, \ldots, \tilde{T} \leq T$ is

$$
\mathbf{I}(0, \tilde{T}) = \mathbf{I}(0,0) * \prod_{t=1}^{\tilde{T}} I(t|t–1), \tag{11}
$$

where $\mathbf{I}(0,0) \equiv 1$. In the same manner that the change in the two-period index is analogous a percent change, the chain index in Equation (11) can be interpreted as the product of percent changes. Consider each two-period increment as $I(t|t–1) = 1 + r_t$ and define $(1 + r_1) * (1 + r_2) * \ldots (1 + r_{\tilde{T}}) = 1 + R_{\tilde{T}}$. Then the chain index can be written as: $\mathbf{I}(0, \tilde{T}) = \mathbf{I}(0, 0)(1 + R_{\tilde{T}})$, which can be written as $R_{\tilde{T}} = \frac{\mathbf{I}(0,\tilde{T})-\mathbf{I}(0,0)}{\mathbf{I}(0,0)}$. From this it can be clearly seen that $R_{\tilde{T}}$ in the percent change in the index from the base period to $\tilde{T}$ (see the section An Introduction to Economic Indices for a numeric example). Two arbitrary time periods of the chain index can be compared by taking the ratio of the chain index at the two periods of interest. For example, the periods $\tilde{T} < \hat{T}$ could be compared as

$$\frac{\mathbf{I}(0,\hat{T})}{\mathbf{I}(0,\tilde{T})} = \prod_{t=\tilde{T}+1}^{\hat{T}} I(t|t–1). \tag{12}$$

Equation (12) shows that the operation of taking a ratio effectively resets the base period. Thus, the relationship between $\tilde{T}$ and $\hat{T}$ has the same percent change interpretation as Equation (11).

The chained class indices of an attribute are constructed by independently chaining the two-period increments of the class index using Equation (11). For the sequence of two-period indices with class j: $I_j(1|0), \ldots, I_j(T|t–1)$, the chained index of class j is $\mathbf{I}_j(0, \tilde{T}) = \prod_{t=1}^{\tilde{T}} I_j(t|t–1)$.

The aggregate chain index also has a decomposition that relates it to the sub-indices of an attribute through the value shares. The decomposition of the chained index differs from the decomposition of the two-period index in a couple of respects. The decomposition expresses the aggregate as a weighted geometric mean (using a first-order approximation) for each class rather than the chained class indices.

Proposition 4. Decomposition of a chained index

> Let J be an attribute as defined above with classes $j \in J$ with sub-indices $I_j(t|t–1)$ for $t = 0, 1, \ldots, T$. The aggregate chained index, $\mathbf{I}(0, \tilde{T})$, (Equation (11)) can be approximately decomposed by class j as product of the value-share weighted geometric mean over time of the sub-index increments $I_j(t|t–1)$

$$\mathbf{I}(T,0) \approx \prod_{j\in J} \prod_{t=1}^{T} I_j(t|t\text{–}1)^{S_j(t|t-1)}. \tag{13}$$

Proof: See Appendix.

Notice however that the value-share weights are (potentially) continually changing over time for a class (e.g., $S_j(1|0) \neq S_j(2|1)$). When the share weights are constant over time $S_j(t|t\text{–}1) = S_j \forall t$ then Equation (34) reduces to

$$\mathbf{I}(T,0) = \prod_j \mathbf{I}_j(T,0)^{S_j}. \tag{14}$$

In this case, the aggregate chain index is also the geometric mean of the chained class indices of an attribute, $\mathbf{I}_j(T,0)$.[10] If the value-share is not constant over time, as will generally be the case, the share-weights for a given class j will not factor. If the share weights are changing substantially over time, particularly when they are systematically trending, the divergence between the product of the class j chained indices and the aggregate chained index can be substantial. In practice, any potential divergence in the between the product of the class chained indices and the aggregate index isn't critically important. The two-period increment to an aggregate chained index has a simple decomposition that relates it to the increments of the sub-indices of an attribute:

$$\mathbf{I}(\tilde{T}+1,0) = \mathbf{I}(\tilde{T},0) * \mathbf{I}(\tilde{T}+1,\tilde{T}) = \mathbf{I}(\tilde{T},0) * \sum_j I_j(T|T-1) * S_j(T|T-1). \tag{15}$$

Since Equation (15) uses only the two-period decompositions this relationship holds for any of the index methods previously considered (Laspeyres, Paasche, or Fisher). Thus, while levels of the Fisher Index may not be easily related through a simple decomposition the marginal/incremental changes have a simple relationship.

The alternative to the chaining indices is a fixed-base index. In contrast, fixed-base indices uses a single (fixed) year as the base for each subsequent (or preceding) period of the index time series. For example, the Laspeyres fixed-base index for period $t \in 0,1,2,\ldots,T$ with base period 0 is $I^L(t|0) = {}^{R(x(t),\,y(0))}/_{R(x(0),\,y(0))}$. The sequence of index values

[10] The relationship between the chained sub-indices and the aggregate chained index holds approximately when share weights are changing trivially or up to a log-normal random error.

$I(1|0), I(2|0), I(3|0), \ldots, I(T|0)$ form the time series of the index. The fixed-base index gives the relative change in the index variable, x, from the base period to the period of interest. If the weighting variable, y, is fairly constant over time then the value-share weights will also be relatively unchanged and there will be little difference in the fixed versus the chain methods.

The fixed-base method of anchoring the base year of the index can make it particularly sensitive to the choice of base year. When there are significant changes or trends in the weighting variable, y, the anchored weights are unable to adjust dynamically as they are with a chained index. Thus, different base years produce different weights and hence different indices. Furthermore, when the data exhibit trends over time the weights being used in periods far from the base may not be representative. To ameliorate this problem, in practice the base year must be periodically moved forward to minimize the bias.

In contrast, chain indices are more robust to changes in the base year because the base change is just a rescaling of the curve. Also, symmetrically-weighted chained (e.g., Fisher) indices are recommended (Hill (1988), Hill (1993), Diewert (2011, Chapter 8)) for data that potentially contain trends. Chain indices also tend to reduce any disparity between the Laspeyres and Paasche indices (Diewert 1978) which bound the "true" economic index. This is an advantage when considering the structural economic interpretation of an index. Chained indices will be equivalent to fixed-base indices when transitivity test is satisfied (Test 8).

Fixed-base indices are slightly easier to interpret and manipulate. First, each period in the time series involves only two periods, which slightly simplifies the comparison of the period of interest to the base and between periods from the chained case (see Equation (12) and the preceding paragraph). Also, the fixed-base method results in a system that is completely additive. This implies that the linear two-period decompositions of are preserved throughout all time periods. Thus, the fixed-base versions of Equations (34) and (15) become simple value-share weighted fixed-base sub-indices à la Equations (7), (8) and (10).

An example of a chained-Fisher Index is the Personal Consumption Expenditures: Chain-type Price Index (PCEPI) (Bureau of Economic Analysis 2012, (1)). The PCEPI is a "market-based" consumer price index that excludes imputed expenditures (Bureau of Economic Analysis 2012, (2)). McCully et al. (2007) compare the PCEPI to the Consumer Price Index (CPI), which is a modified Laspeyres fixed-base index, to identify the relative weight of sources of discrepancy. Controlling for temporal aggregation and scope (the baskets of goods) they find that the formula (chained-Fisher vs. fixed-Laspeyres) used accounted for a significant portion of the difference between the two indices.

The degree to which the weighting variable y is changing directly affects the decision to use a fixed-base versus a chained index. When data (e.g., prices and quantities) in consecutive periods are *more similar* the weighting variable changes more slowly and potential biases from unrepresentative weighting will be mitigated (Allen and Diewert 1981). The similarity-dissimilarity notion is captured between the Laspeyres and Paasche indices, which use $y(0)$ and $y(t)$, respectively. Hill (1997) proposes comparing the measures $\max({}^{I^L}/_{I^A}, {}^{I^A}/_{I^L})$ for the chain and fixed indices, where smaller values indicate the preferred method. While Diewert (2011, Chapter 8) notes that this measure is imperfect, where differences are marked the preference should be clear.

Empirical Example: Economic Indices Gulf of Alaska Shoreside Sector

This empirical example will display the index methods previously developed in previous sections and how they can be implemented. The focus will be on commercial fisheries in the Gulf of Alaska (GOA) which can be broken up into two sectors: the at-sea fleet(which is comprised of catcher-processors and motherships that have on-board processing facilities) and the shoreside sector (in which catcher-vessels typically deliver their harvest to shoreside processors). The indices presented here will cover the GOA-shoreside fleet; these complete suite for all sectors in Alaska can be found in the Economic SAFE report (Hiatt et al. 2011). There are two markets in the shoreside sector: the first occurs “at the dock” where catcher-vessels deliver harvest to processors,[11] which is known as the ex-vessel market; the second is the first-wholesale (hereafter simply “wholesale”) market, where processors sell products on the national/global market. Indices for these two markets are analyzed separately, and for each market we consider two attribute dimensions. In the ex-vessel market the attributes con-

[11]Delivery of harvest is considered production by the harvest sector. The word “production” will be used generically to refer to delivered catch or the production of processed fisheries goods from the wholesale sector.

sidered will be the species type and gear type used to capture the harvest brought to market. First-wholesale market goods are characterized along two attribute dimensions: species type and product type. Table 1 lists the species and product types considered in the analysis.

Data

Calculating economic indices requires data on production, revenues, and prices. Wholesale and ex-vessel production and revenue data were obtained from NMFS Alaska Region's Catch-Accounting System (CAS), Weekly Production Reports (WPR), Alaska Department of Fish and Game (ADF&G), and the Commercial Operator's Annual Report (COAR). Most data are housed and maintained by the Alaska Fisheries Information Network (AKFIN), the exception being at-sea production data which are managed by the NMFS Alaska Region. Economic production and revenue data for the years 2000-2010 used in the analysis are compiled and maintained by the Alaska Fisheries Science Center (AFSC) Economics and Social Sciences Research Program (ESSRP) and AKFIN for the Economic SAFE.[12]

Products were grouped into 11 distinct product types in Table 1. Species were classified into 7 species or species complexes in Table 1. The classification of products and species complex are identical to the methods used in the current and previous the Economic SAFE. Wholesale production and revenue data were summed up to species and product type. Ex-vessel data were summed up to species and gear-type. Prices (p) were calculated by dividing revenue in U.S. dollars (v) by pound of production (q), $p = {}^{v}\!/_{q}$. Prices, revenues and indices were calculated in nominal terms.

The Application of Index Methods

This methods section operationalizes the chained-Fisher indices developed in the previous sections and their decomposition along attribute dimensions. Specifically, let Ω be the set of all of species and products in the wholesale market

(e.g., $\{cod\ fillets, pollock\ fillets, \ldots, pollock\ roe, \ldots\}$), or species and gear-types in the ex-vessel market

[12]These data were obtained from the identical sources used to report data for the Economic SAFE. Details for the validation and aggregation of the data can be obtained by contacting ESSRP Ben.Fissel@NOAA.gov.

(e.g., $\{trawl\ caught\ cod, hook\text{–}and\text{–}line\ caught\ cod, \ldots, trawl\ caught\ pollock, \ldots\}$). Let $\omega \subseteq \Omega$ be the subset of goods sharing a class for a given attribute (see the introduction and methods section above for a more precise description of classes and attributes). The aggregate index is represented in this notation by setting $\omega = \Omega$. The two-period value, price and quantity indices for the class ω over the years $t \in$ 2001–2010 are[13]

$$\begin{aligned}
V^*_\omega(t|t\text{–}1) &= \frac{\sum_{i\in\omega} p_i(t)q_i(t))}{\sum_{i\in\omega} p_i(t\text{–}1)q_i(t\text{–}1)} \\
P^*_\omega(t|t\text{–}1) &= \sqrt{\frac{\sum_{i\in\omega} p_i(t)q_i(t\text{–}1))}{\sum_{i\in\omega} p_i(t\text{–}1)q_i(t\text{–}1)} \frac{\sum_{i\in\omega} p_i(t)q_i(t))}{\sum_{i\in\omega} p_i(t\text{–}1)q_i(t)}} \\
Q^*_\omega(t|t\text{–}1) &= \sqrt{\frac{\sum_{i\in\omega} q_i(t)p_i(t\text{–}1))}{\sum_{i\in\omega} q_i(t\text{–}1)p_i(t\text{–}1)} \frac{\sum_{i\in\omega} q_i(t)p_i(t))}{\sum_{i\in\omega} q_i(t\text{–}1)p_i(t)}}.
\end{aligned} \tag{16}$$

The year $T \in$ 2001–2010 chained indices are then calculated as

$$\begin{aligned}
\mathbf{V}_\omega(2001, T) &= \prod_{t=2001}^{T} V^*_\omega(t|t\text{–}1) \\
\mathbf{P}_\omega(2001, T) &= \prod_{t=2001}^{T} P^*_\omega(t|t\text{–}1) \\
\mathbf{Q}_\omega(2001, T) &= \prod_{t=2001}^{T} Q^*_\omega(t|t\text{–}1).
\end{aligned} \tag{17}$$

Indices were set to a base year of 2006 by dividing through by the year 2006 price index, $\mathbf{V}_\omega(2006, T) = {}^{\mathbf{V}_\omega(2001,T)}/_{\mathbf{V}_\omega(2001,2006)}$, $\mathbf{P}_\omega(2006, T) = {}^{\mathbf{P}_\omega(2001,T)}/_{\mathbf{P}_\omega(2001,2006)}$ and $\mathbf{Q}_\omega(2006, T) = {}^{\mathbf{Q}_\omega(2001,T)}/_{\mathbf{Q}_\omega(2001,2006)}$.

The attribute indices have been color coded (Figs. 1-4) to indicate changes that influence the aggregate index. The influence that incremental changes in the sub-indices have on the aggregate index is a function of both the value share and the magnitude of change in a sub-index (see the section Relating Disaggregate and Aggregate Economic Indices for details). Thus, the color coding of the indices is determined by calculating the percent change in the sub-index multiplied by the current period value share. The value shares at time t were calculated as proportion of value from goods in ω (the goods in a class) to the value of goods in

[13]Data for the year 2000 was available and used to create the 2001 two-period index.

Ω (the aggregate market), $S_\omega(t) = \sum_{i\in\omega} v_i(t) / \sum_{i\in\Omega} v_i(t)$. [14] The change in the index is given by the two-period increment to the index minus one, $\%\Delta\mathbf{V}_\omega(2006, t) \approx (V^*_\omega(t|t\text{–}1) - 1)$, $\%\Delta\mathbf{P}_\omega(2006, t) \approx (P^*_\omega(t|t\text{–}1) - 1)$ and $\%\Delta\mathbf{Q}_\omega(2006, t) \approx (Q^*_\omega(t|t\text{–}1) - 1)$ (Equations (31) and (27)). Thus, the measures of the influence of the sub-indices on the aggregate index are $\%\Delta\mathbf{V}_\omega(2006, t) * S_\omega$, $\%\Delta\mathbf{Q}_\omega(2006, t) * S_\omega$, and $\%\Delta\mathbf{Q}_\omega(2006, t) * S_\omega$. Red indicates the influence measure is < -0.1, blue indicate no impact and green indicates an influence measure > 0.1. These color coding threshold values were subjectively chosen to visually contrast distinct changes in the indices.

Economic Indices for Gulf of Alaska Shoreside Fisheries

The index figures in (Figs. 1-4) are designed to help the reader visualize changes in the indices and relate the changes to shifts in aggregate value, prices, and quantities. Each of the GOA shoreside markets have two figures each presenting the indices along the two separate attribute dimensions: species and gear type for the ex-vessel market (Figs. 1-2), and species and product type for the wholesale market (Figs. 3-4). The figures are broken up into panels with the aggregate index in the upper-left panel, the sub-indices of an attribute in the lower-left panel, and supporting plots of the contemporaneous value share in the upper-right panel and a bar graph that decomposes annual production (100,000 metric tons) of the classes by the other attribute dimension in that market (e.g., production of species by gear type). In the aggregate and sub-indices panels along the rows are value, price, and quantity indices (Equation (17)) allowing the comparison of price and quantity changes to total value. Along the columns of the sub-index panel are the different classes which can be compared to the aggregate index above. The color coding of the sub-indices along with the value-share plot in upper-right panel allow comparison of changes in the different sub-indices to changes in the aggregate index. The bar graphs of annual production both convey the relative production levels but also help relate indices across figures within the same market (Fig. 1 to Fig. 2 and

[14] In theory the weights that relate the Fisher sub-indices to the aggregate index are a combination of the Laspeyres and Paasche value-shares (Equation (32)). For color coding and reporting the contemporaneous ($S_\omega(t)$) value-share weights were used to avoid confusion between the reported value share and the theoretical weights S^F (Equation (32)). Since the current period value share was only used for color coding and the value share between consecutive periods is generally small there is no practical difference.

Fig. 3 to Fig. 4).

The following sections analyze the Gulf of Alaska shoreside markets to provide an example of the information that can be garnered from the economic indices of an attribute. The analysis and text in these sections is the same as what can be found in the economic chapter of the SAFE (Hiatt et al. 2011). Additionally, the SAFE contains analysis of the GOA at-sea, BSAI shoreside, and BSAI at-sea markets.

Analysis of the Gulf of Alaska Shoreside Ex-vessel Market

Value from deliveries is largely concentrated in three key species: sablefish, cod, and pollock (upper-right panel Figs. 1-2). In 2010, 50% of the value came from sablefish, just under 20% from pollock, and just over 25% from Pacific cod. The hook-and-line gear type, used to harvest of sablefish and to a lesser extent cod, accounts for over half of the value share. Deliveries of fish caught using pot gear account for 13% of the value and trawl gear accounts for 32% of the value.

Composition bar graphs show that with the exception of cod, species are primarily harvested with a single gear type (lower-right panel Fig. 1). The composition of gear types for cod shows that pot gear (which is only used in cod) has become increasingly important. Increased cod harvests have largely come from catcher vessels using pot gear, although hook-and-line deliveries have increased some as well. Trawl gear still accounts for roughly two-thirds of the total quantities (weight) delivered to processors as a result of the larger pollock and cod harvests. The implied difference between value and catch is relative price of species targeted and caught using the different gears. The aggregate quantity index increased by 18% in 2010 (left panels Figs. 1-2). Species quantity indices indicate that increase was due to significant increases in pollock and cod harvests. The pollock quantity index, which had been steadily falling over the last 5 years, increased significantly in 2010 returning the index to just over 2006 levels. Cod has been fairly stable over the last 5 years despite a slight decrease in deliveries in 2009. The sablefish quantity index has been gradually declining since 2005 as a result of decreased TAC. Commensurate with the decline in sablefish, hook-and-line quantity index has fallen as well. The comparatively constant, and even increasing, delivered weight from hook-and-line vessels (lower-right panel Fig. 2) show that effort from this sector has been redirected to the lower-priced cod. The trawling sector quantity index has been relatively stable compared to the declining pollock index, the species that it primarily targets. The increased harvest of flatfish indicate that the trawling sector has shifted effort toward these species as the availability of pollock fell.

The aggregate price index rose 6% in 2010 from 107 to 113 (left panels Figs. 1-2). Over the last 5 years, the aggregate price index for GOA shoreside catches rose peaking at 125 in 2008. In 2009 the price dropped significantly (14%) some of which was made up by the 2010 price increase. With the aggregate price index at 113, 2010 prices are still well above 2006 base year for the index. Price increases have been driven primarily by increases in the sablefish price index. Changes in the cod price index have contributed significantly to the observed aggregate price variation. The 2008 drop in prices came mostly from a drop in cod price index. The high correspondence between the price index for the wholesale market (Fig. 3) and the ex-vessel price index indicates efficient an ex-vessel market in which wholesale prices effectively pass through from one market to the other.

Increasing quantities and increasing price resulted in a significant increase (25%) in the aggregate value index for 2010 (left panels, Figs. 1-2). Over the last 5 to 7 years, the steady rise in the price index and low volatility in the quantity index have translated to an upward trending value index. The 2010 increase in the aggregate value index increases was driven by two factors. First, increases in the cod and pollock value indices were the result of quantity index increases from these species. Second, sablefish value increased due to the rising price. In terms of ex-vessel value, sablefish accounts for a noticeably larger share of the value in the sector. Reductions in cod catches together with the change in price for delivered cod resulted in a drop in the aggregate value in 2009. Gear type value indices show that the aggregate gains in value (and loss in 2008) has been experienced by all gear types.

Despite a significant drop in 2009, the stability of sablefish as well as increases in cod catch resulted in a return of aggregate ex-vessel value to near its 2008 levels. The long-run increases and generally steady increase in value has resulted 56% increase in value since 2001. Steady value growth in light of TAC reductions in pollock and sablefish was achieved by a shift to flatfish by the trawl sector, the shift to cod by the hook-and-line sector, and sablefish price increases.

Analysis of the Gulf of Alaska Shoreside Wholesale Market

Because the delivery of catch feeds production and sales to the wholesale market, trends in the GOA shoreside wholesale sector largely mirror the ex-vessel market. The market is primarily focused on sablefish, cod, pollock and to a much lesser extent rockfish and flatfish (top-right panel, Figs. 3-4). As a proportion of total value in 2010 sablefish accounted for 31%, cod 35%, and pollock 25%. Flatfish and rockfish collectively made up 9% of total value. The shoreside market is fairly diversified in value although the high correlation between changes in the pollock and cod market factor (prices and quantities) increases the sector's exposure. Across product types, value largely comes from head-and-gut products (48%) and fillets (28%) with the remaining value distributed across a variety of product types.

In contrast to the ex-vessel market where sablefish has larger value share (50%), sablefish only accounts for 30% of the wholesale value share. Quantities in the ex-vessel and wholesale markets must be roughly equal, (differing only due to product recovery rates). Difference in relative value share between the markets must come from the relative price per pound of the three primary species: sablefish, cod, and pollock. Relative price differences are in-part attributable to the amount of post-harvest processing done by processors. Sablefish is largely processed as a head-and-gut product, while other species are made into products with higher value added such as fillets, and surimi. The capitalization of this expense in the final product price changes the relative species price.

Composition bar graphs show that for most species, output is distributed across a variety of product types (lower-right panel Figs. 3-4). In particular, pollock, cod, flatfish, and rockfish production is balanced across fillets, head-and-gut, surimi, whole fish, and "other" product types.[15] Sablefish is the exception and is highly concentrated in head-and-gut products. Because pollock and cod are the major species processed shoreside they make up disproportionate shares of the species composition by product type. Surimi and roe both come almost

[15] The "other" product type typically consists of ancillary products such as heads, stomachs, etc.

entirely from pollock. Fillets are basically either pollock or cod. In contrast, head-and-gut and whole fish production are balanced across species.

The aggregate quantity index was up over 30% in 2010, indicating a significant jump in the amount of product produced and sold on the wholesale market. The jump comes after iterative declines over the previous 5 years. The 2010 quantity index increase was the result of significant increases in the cod and pollock indices, and to a lesser extent rockfish. The sablefish quantity index has been trending down particularly over the last 5 to 7 years as the GOA sablefish TAC has been reduced. Until 2010, pollock output has also been trending down also as a result of a decreasing TAC. The sizable increase in flatfish and rockfish production over roughly the same period indicates that effort shifted to producing products from these species as abundances of pollock and sablefish decreased.

The aggregate price index increased 4% in 2010 from 108 to 112 (left panels Figs. 3-4)). Similar to the price dynamics in other regions focused in pollock and cod, the 2010 price index increase comes after a marked drop in prices in 2009 which was in turn preceded by a price spike in 2008. The drop in 2009 thus appears to be a reversion of prices to the upward trend in prices earlier in the decade. Species price indices show that the 2010 price increase was primarily driven by increases in the price of sablefish (15%) whose prices has been increasing over the last decade. The increase in 2008 prices and corresponding drop in 2009 prices were driven largely by the fall in cod and pollock prices during which time sablefish price increases were more modest. As with other regions, pollock and cod price increases in 2008 were associated with an increase in the surimi price index. Changes in the fillet and head-and-gut price indices were the primary drivers in the subsequent aggregate price index changes. As production is diversified across a variety of product types, these indices' prices don't point to a single product type that is driving the variation in aggregate prices over the long run.

The simultaneous increase in both the price and quantity indices in 2010 resulted in a 37% increase in the aggregate value index (left panels Figs. 3-4). This erased the equally large drop in value that had occurred the previous year. The dramatic changes in value are primarily the result of value changes in cod, and pollock. The increase in cod and pollock production in 2010 and the leveling off of prices for these key species has resulted in a return of the value index to its 2008 levels. Sablefish and rockfish value changes contributed as well, particularly in the 2010 value index increase.

Looking at the longer time horizon, we see that the aggregate value index in the GOA shoreside wholesale sector is well above the 2001 level. With the value index at 119 in 2010 and 76 in 2001, this translates into a 55% increase in wholesale value over the last decade. Diversification across product types and species has contributed. Shoreside production and value is still somewhat concentrated in pollock and cod, and hence sensitive to changes in the availability and price of these species. However, the significance of sablefish in value share and the observed ability to shift production to flatfish and rockfish has help buffer the shoreside market. The correspondence of index trends and variation in price and value indicates an effective link between ex-vessel and wholesale markets in which prices passes through from one market to the other.

Conclusion

Standard economic indices can provide insight into characterizing both the aggregate and disaggregate behavior of economic variables. These indices provide market participants and fisheries management with a broad perspective on aggregate performance while simultaneously characterizing and simplifying the myriad of influencing factors into a digestible format that is useful for quickly analyzing changes in a fishery. The perspective provided by these indices is useful for considering both the retrospective impact of management decisions as well as informing future policy. Indices are visually presented in a manner that is intended to facilitate the understanding of the relative influence that various species, product types, and gear types carry in determining aggregate behavior.

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Table 1. -- Attributes and classes.

Abbreviation	Class description
Species Attribute Classes	
AMCK	Atka mackerel
FLAT	Flatfish complex
OTHR	Skate complex
PCOD	Pacific cod
PLCK	Walleye pollock
ROCK	Rockfish complex
SABL	Sablefish (black cod)
Gear-Type Attribute Classes	
Abbreviation	Class description
TWL	Trawl gear
HAL	Hook-and-line
POT	Pot gear
Product Attribute Classes	
Abbreviation	Class description
Whole	Whole fish: food fish; processed for bait; bled only.
Head & Gut	Headed and gutted: with roe; western cut; eastern cut; tail removed. Gutted and head on.
Kirimi	Headed, gutted and kirimi cut
Salt & Split	Headed, gutted, salted, and split
Roe	Roe: loose or in sacs or skeins.
Fillet	Fillets: skin and ribs; no ribs; ribs and no skin; skinless/boneless.
Deep-Skin	Fillets: deep-skin.
Surimi	Surimi: paste from fish.
Minced	Minced and ground flesh.
Meal	Fish meal: whole fish; other meal
Other	Wings; stomachs; mantles.

Table 2. -- Species indices and value share for the Gulf of Alaska shoreside ex-vessel market.

	Species	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Value Index	Aggregate	74.12	61.67	82.13	85.07	88.75	100.00	105.40	122.74	92.46	115.49
Price Index	Aggregate	81.70	75.18	83.03	74.56	82.31	100.00	111.07	124.60	107.07	112.96
Quantity Index	Aggregate	90.73	82.03	98.92	114.10	107.82	100.00	94.89	98.50	86.36	102.24
Value Index	FLAT	56.76	41.56	31.60	29.23	56.26	100.00	112.93	129.70	105.63	74.35
Price Index	FLAT	115.93	89.14	83.24	60.99	84.40	100.00	110.22	102.27	95.56	77.18
Quantity Index	FLAT	48.96	46.62	37.96	47.94	66.66	100.00	102.46	126.82	110.54	96.33
Value Share	FLAT	0.04	0.03	0.02	0.02	0.03	0.05	0.05	0.05	0.05	0.03
Value Index	PCOD	53.77	45.78	78.62	81.13	80.32	100.00	133.18	156.71	77.43	107.14
Price Index	PCOD	72.05	65.52	76.15	67.58	74.51	100.00	125.94	133.22	74.72	66.95
Quantity Index	PCOD	74.63	69.88	103.25	120.04	107.80	100.00	105.75	117.64	103.63	160.03
Value Share	PCOD	0.19	0.20	0.25	0.25	0.24	0.27	0.34	0.34	0.22	0.25
Value Index	PLCK	96.08	57.07	49.15	67.16	104.49	100.00	77.79	92.97	74.11	136.64
Price Index	PLCK	94.11	78.78	70.03	75.24	91.67	100.00	106.89	134.13	128.49	127.85
Quantity Index	PLCK	102.08	72.45	70.19	89.25	113.98	100.00	72.77	69.31	57.67	106.87
Value Share	PLCK	0.20	0.14	0.09	0.12	0.18	0.16	0.12	0.12	0.13	0.19
Value Index	ROCK	96.18	75.45	76.22	71.88	75.75	100.00	114.73	111.81	67.17	92.43
Price Index	ROCK	54.26	57.01	58.97	63.67	77.50	100.00	103.05	105.77	70.22	84.03
Quantity Index	ROCK	177.24	132.34	129.24	112.89	97.73	100.00	111.32	105.71	95.65	110.00
Value Share	ROCK	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.02	0.02
Value Index	SABL	79.34	73.45	99.10	99.50	92.70	100.00	97.79	112.62	105.54	117.74
Price Index	SABL	83.30	79.30	90.12	78.68	83.55	100.00	103.84	120.67	129.14	151.23
Quantity Index	SABL	95.24	92.63	109.96	126.46	110.95	100.00	94.18	93.33	81.72	77.86
Value Share	SABL	0.53	0.58	0.59	0.57	0.51	0.49	0.46	0.45	0.56	0.50

Table 3. -- Gear indices and value share for the Gulf of Alaska shoreside ex-vessel market.

	Gear	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Value Index	Aggregate	74.12	61.67	82.13	85.07	88.75	100.00	105.40	122.74	92.46	115.49
Price Index	Aggregate	81.70	75.18	83.03	74.56	82.31	100.00	111.07	124.60	107.07	112.96
Quantity Index	Aggregate	90.73	82.03	98.92	114.10	107.82	100.00	94.89	98.50	86.36	102.24
Value Index	HAL	80.14	73.85	97.37	98.35	91.33	100.00	100.79	116.46	104.99	116.80
Price Index	HAL	82.52	79.03	89.28	78.44	83.18	100.00	106.47	120.86	120.24	137.16
Quantity Index	HAL	97.11	93.45	109.07	125.38	109.80	100.00	94.67	96.36	87.31	85.15
Value Share	HAL	0.58	0.65	0.64	0.62	0.56	0.54	0.52	0.51	0.61	0.55
Value Index	POT	17.13	19.33	66.56	71.48	76.94	100.00	123.62	151.38	69.56	101.29
Price Index	POT	73.18	64.22	75.73	65.93	74.38	100.00	122.29	148.80	74.47	68.07
Quantity Index	POT	23.40	30.10	87.89	108.42	103.44	100.00	101.09	101.73	93.42	148.81
Value Share	POT	0.03	0.05	0.12	0.13	0.13	0.15	0.18	0.19	0.11	0.13
Value Index	TWL	91.32	60.99	63.09	68.50	89.98	100.00	104.57	119.78	81.73	120.11
Price Index	TWL	83.33	71.79	73.65	71.10	84.57	100.00	113.52	119.03	101.21	97.20
Quantity Index	TWL	109.59	84.96	85.66	96.34	106.40	100.00	92.12	100.63	80.76	123.57
Value Share	TWL	0.38	0.31	0.24	0.25	0.31	0.31	0.31	0.30	0.27	0.32

Table 4. -- Species indices and value share for the Gulf of Alaska shoreside wholesale market.

	Species	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Value Index	Aggregate	76.48	72.10	73.36	81.05	99.75	100.00	104.81	117.14	86.99	118.75
Price Index	Aggregate	86.89	89.45	86.97	86.07	100.81	100.00	106.73	123.43	107.86	112.40
Quantity Index	Aggregate	88.02	80.60	84.35	94.17	98.96	100.00	98.19	94.91	80.65	105.64
Value Index	FLAT	48.94	45.76	31.86	29.81	64.92	100.00	112.50	131.85	98.68	72.60
Price Index	FLAT	100.24	118.12	115.58	82.90	116.02	100.00	104.44	102.47	90.55	95.17
Quantity Index	FLAT	48.82	38.75	27.57	35.96	55.95	100.00	107.72	128.68	108.98	76.28
Value Share	FLAT	0.05	0.04	0.03	0.03	0.05	0.07	0.08	0.08	0.08	0.04
Value Index	PCOD	67.87	79.10	74.15	87.20	91.49	100.00	132.33	150.05	87.95	131.94
Price Index	PCOD	77.07	78.22	72.53	81.24	88.16	100.00	114.88	123.41	85.59	83.21
Quantity Index	PCOD	88.06	101.13	102.24	107.34	103.78	100.00	115.19	121.59	102.75	158.57
Value Share	PCOD	0.27	0.33	0.31	0.32	0.28	0.30	0.38	0.39	0.31	0.34
Value Index	PLCK	93.80	54.28	49.01	67.14	124.02	100.00	71.34	87.05	62.02	118.10
Price Index	PLCK	92.60	87.33	84.87	93.90	118.98	100.00	98.83	153.95	142.51	129.72
Quantity Index	PLCK	101.30	62.15	57.74	71.50	104.23	100.00	72.19	56.54	43.52	91.04
Value Share	PLCK	0.30	0.19	0.17	0.21	0.31	0.25	0.17	0.18	0.18	0.25
Value Index	ROCK	53.34	95.17	60.78	53.36	99.30	100.00	97.64	89.63	73.67	106.55
Price Index	ROCK	86.37	104.11	90.02	71.44	100.00	100.00	100.81	90.90	84.04	94.37
Quantity Index	ROCK	61.76	91.42	67.52	74.69	99.31	100.00	96.86	98.61	87.66	112.90
Value Share	ROCK	0.04	0.07	0.05	0.04	0.06	0.06	0.05	0.04	0.05	0.05
Value Index	SABL	82.99	82.39	102.07	103.34	97.61	100.00	104.24	110.26	104.14	117.47
Price Index	SABL	86.64	92.19	94.39	84.57	96.35	100.00	104.31	120.97	128.49	153.16
Quantity Index	SABL	95.79	89.37	108.13	122.20	101.30	100.00	99.93	91.15	81.05	76.70
Value Share	SABL	0.34	0.36	0.44	0.40	0.31	0.32	0.31	0.30	0.38	0.31

Table 5. -- Product indices and value share for the Gulf of Alaska shoreside wholesale market.

	Product	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Value Index	Aggregate	76.48	72.10	73.36	81.05	99.75	100.00	104.81	117.14	86.99	118.75
Price Index	Aggregate	86.89	89.45	86.97	86.07	100.81	100.00	106.73	123.43	107.86	112.40
Quantity Index	Aggregate	88.02	80.60	84.35	94.17	98.96	100.00	98.19	94.91	80.65	105.64
Value Index	Fillet	67.37	82.86	82.19	60.41	90.64	100.00	95.64	111.59	96.24	131.18
Price Index	Fillet	62.76	74.00	70.73	68.92	90.57	100.00	112.34	120.91	92.95	91.69
Quantity Index	Fillet	107.35	111.98	116.20	87.65	100.08	100.00	85.13	92.29	103.54	143.08
Value Share	Fillet	0.22	0.29	0.29	0.19	0.23	0.26	0.23	0.24	0.28	0.28
Value Index	Head&Gut	67.32	59.23	70.46	91.48	97.39	100.00	116.97	119.50	91.38	123.25
Price Index	Head&Gut	84.09	94.56	93.55	85.42	99.15	100.00	105.28	119.80	112.92	125.62
Quantity Index	Head&Gut	80.05	62.64	75.32	107.10	98.22	100.00	111.10	99.75	80.93	98.12
Value Share	Head&Gut	0.41	0.38	0.44	0.52	0.45	0.46	0.51	0.47	0.48	0.48
Value Index	Other	75.95	105.93	93.06	93.26	100.49	100.00	91.13	153.01	101.01	109.55
Price Index	Other	118.73	86.69	76.25	103.15	106.77	100.00	105.12	118.49	113.53	110.44
Quantity Index	Other	63.97	122.19	122.05	90.42	94.11	100.00	86.68	129.13	88.97	99.20
Value Share	Other	0.09	0.14	0.12	0.11	0.10	0.10	0.08	0.12	0.11	0.09
Value Index	Roe	189.21	68.28	63.17	91.07	160.91	100.00	91.88	78.40	38.03	41.45
Price Index	Roe	172.60	118.32	120.76	175.81	170.75	100.00	85.92	126.47	117.28	67.95
Quantity Index	Roe	109.62	57.71	52.31	51.80	94.24	100.00	106.93	61.99	32.42	61.00
Value Share	Roe	0.13	0.05	0.05	0.06	0.09	0.05	0.05	0.04	0.02	0.02
Value Index	Surimi	112.15	96.48	69.56	80.28	142.09	100.00	75.02	136.07	58.30	159.87
Price Index	Surimi	77.44	86.17	82.30	80.44	103.83	100.00	102.34	214.85	158.21	168.71
Quantity Index	Surimi	144.82	111.96	84.52	99.80	136.85	100.00	73.30	63.33	36.85	94.76
Value Share	Surimi	0.09	0.08	0.06	0.06	0.09	0.06	0.04	0.07	0.04	0.08
Value Index	Whole	67.97	75.88	73.54	55.64	65.92	100.00	137.81	110.95	111.62	109.14
Price Index	Whole	83.78	90.94	100.09	79.48	92.27	100.00	142.40	108.56	94.10	94.05
Quantity Index	Whole	81.13	83.44	73.48	70.00	71.44	100.00	96.78	102.20	118.61	116.05
Value Share	Whole	0.04	0.04	0.04	0.03	0.03	0.04	0.05	0.04	0.05	0.04

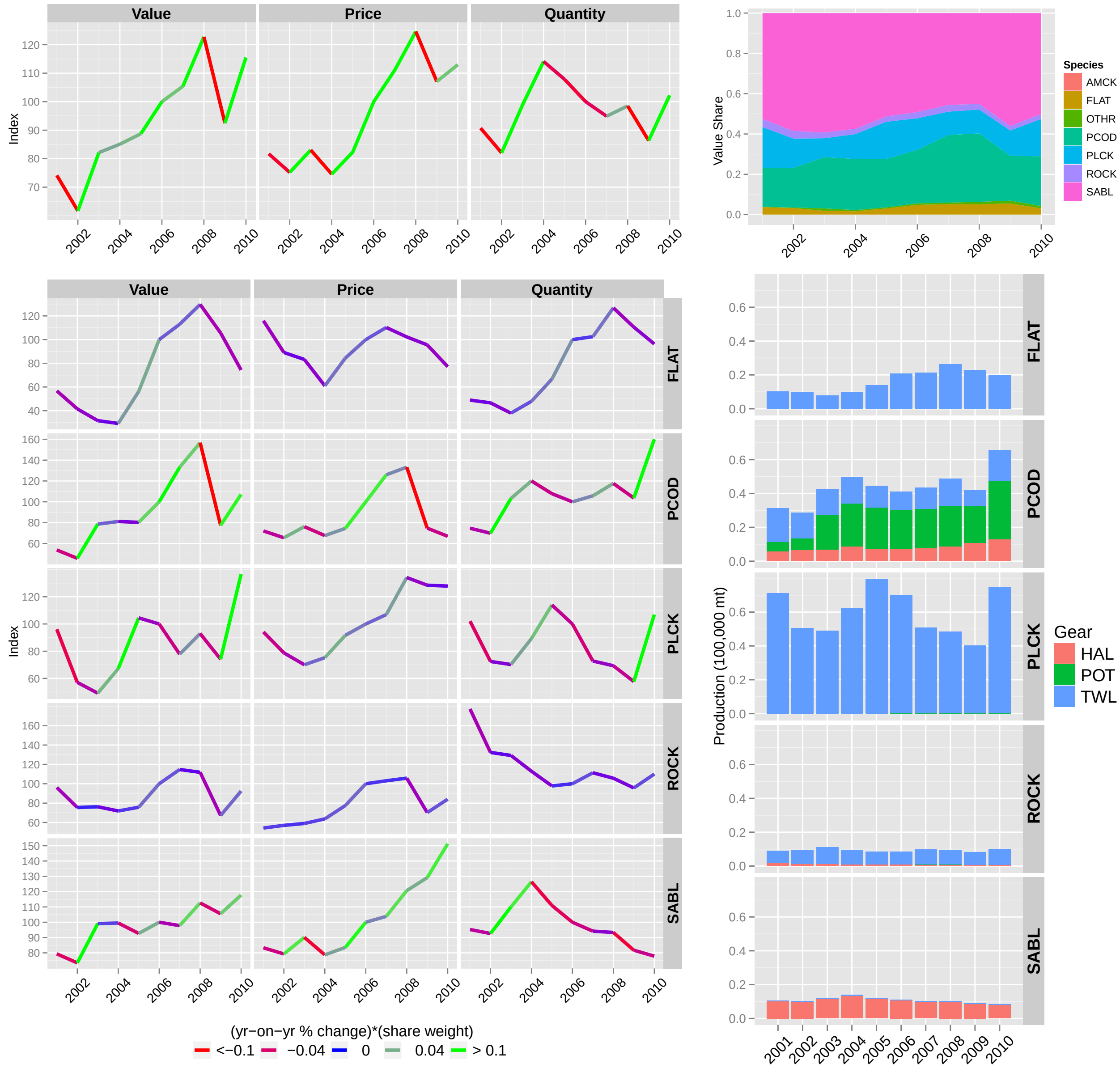

Figure 1. -- Gulf of Alaska shoreside ex-vessel market: species decomposition 2001−2010. upper-left panel: Aggregate value, price and quantity indices, base(2006)=100
lower-left panel: Species attribute indices (rows) of value, price and quantity indices (columns), base(2006)=100
upper-right panel: Annual contemporaneous value share by species
lower-right panel: Annual production (100,000 metric tons) of species by gear type

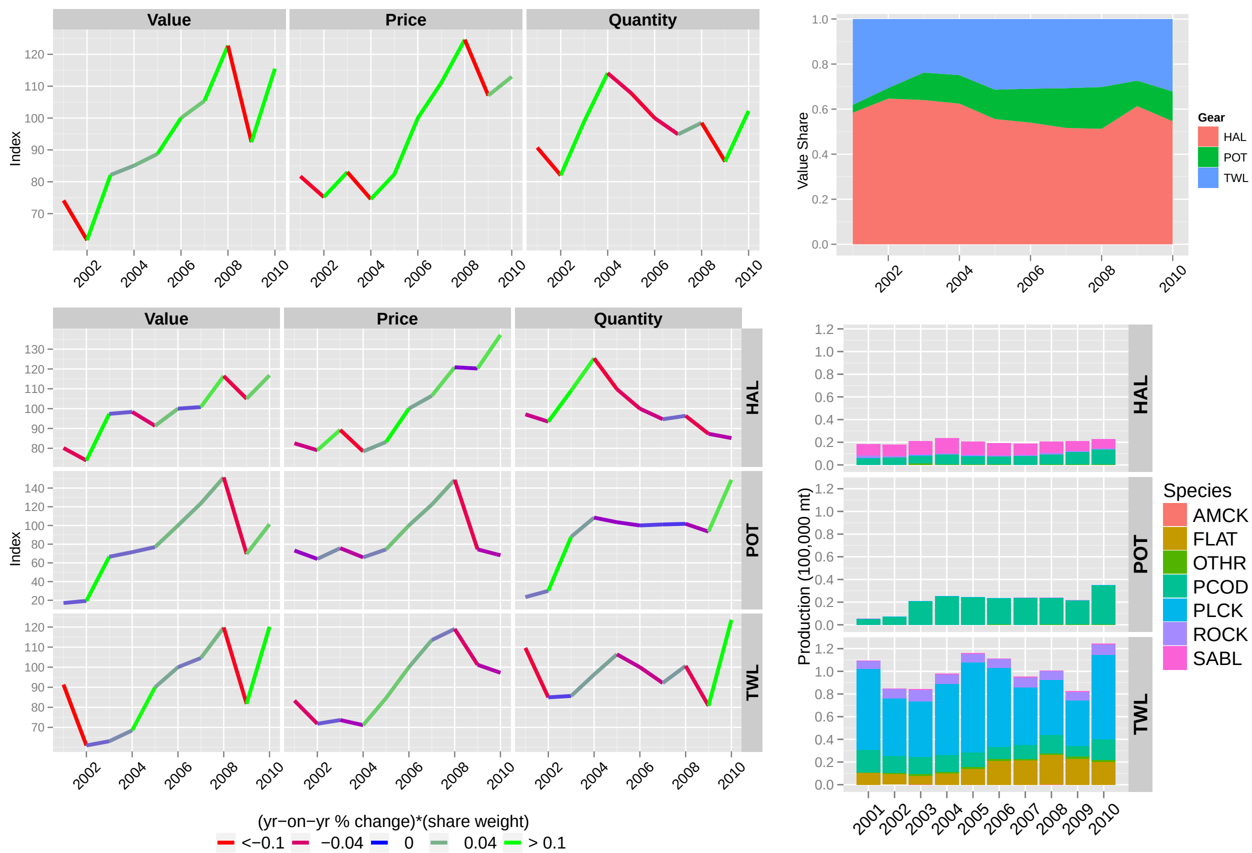

Figure 2. -- Gulf of Alaska shoreside ex-vessel market: gear type decomposition 2001−2010. upper-left panel: Aggregate value, price and quantity indices, base: base(2006)=100

lower-left panel: Gear-type attribute indices (rows) of value, price and quantity indices (columns), base(2006)=100

upper-right panel: Annual contemporaneous value share by gear-type

lower-right panel: Annual production (100,000 metric tons) of gear-types by species

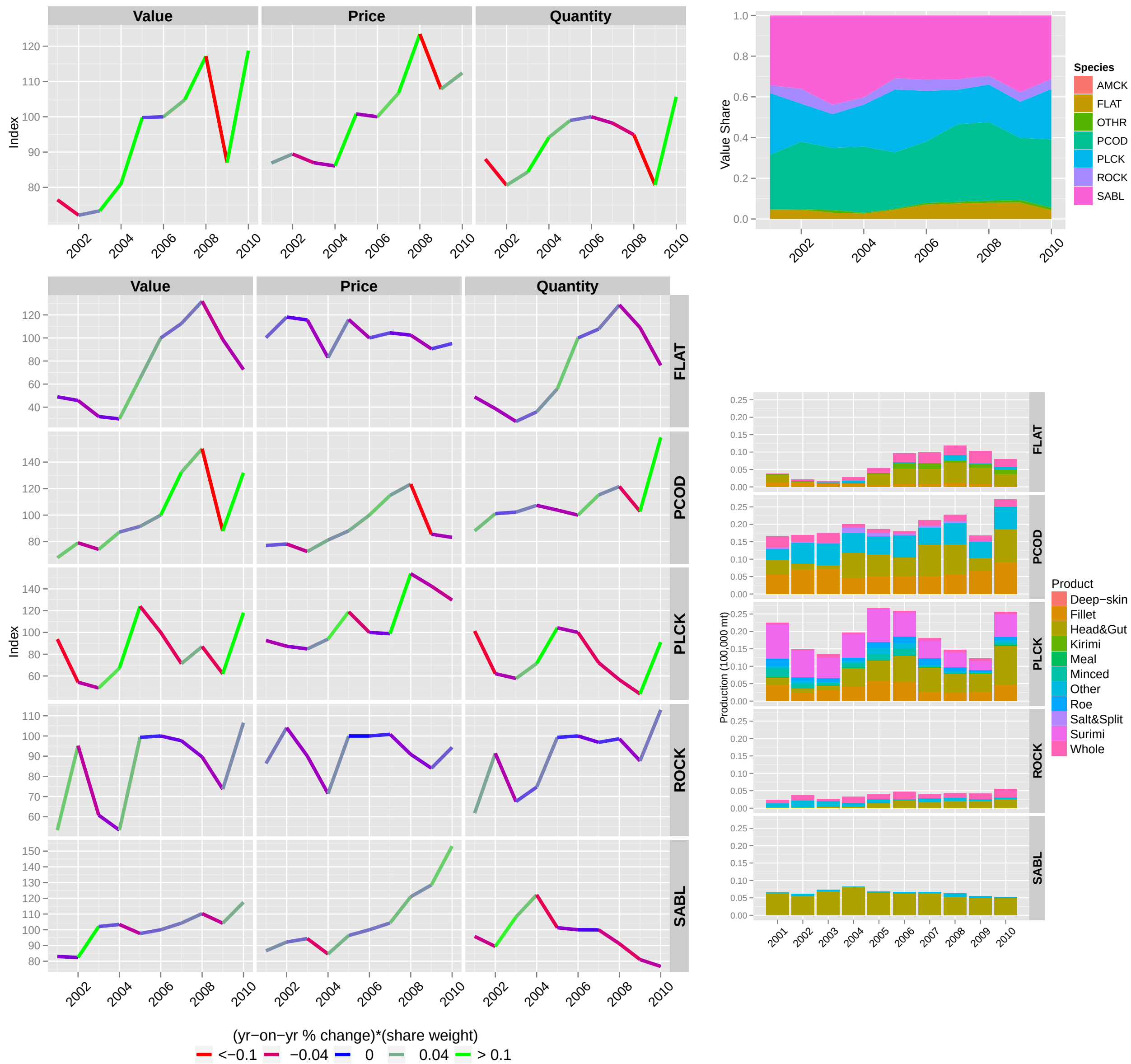

Figure 3.- - Gulf of Alaska shoreside first-wholesale market: species decomposition $2001 - 2010$.
upper-left panel: Aggregate value, price and quantity indices, $\mathrm{base}(2006){=}100$
lower-left panel: Species attribute indices (rows) of value, price and quantity indices (columns), $\mathrm{base}(2006){=}100$
upper-right panel: Annual contemporaneous value share by species
lower-right panel: Annual production (100,000 metric tons) of species by product type

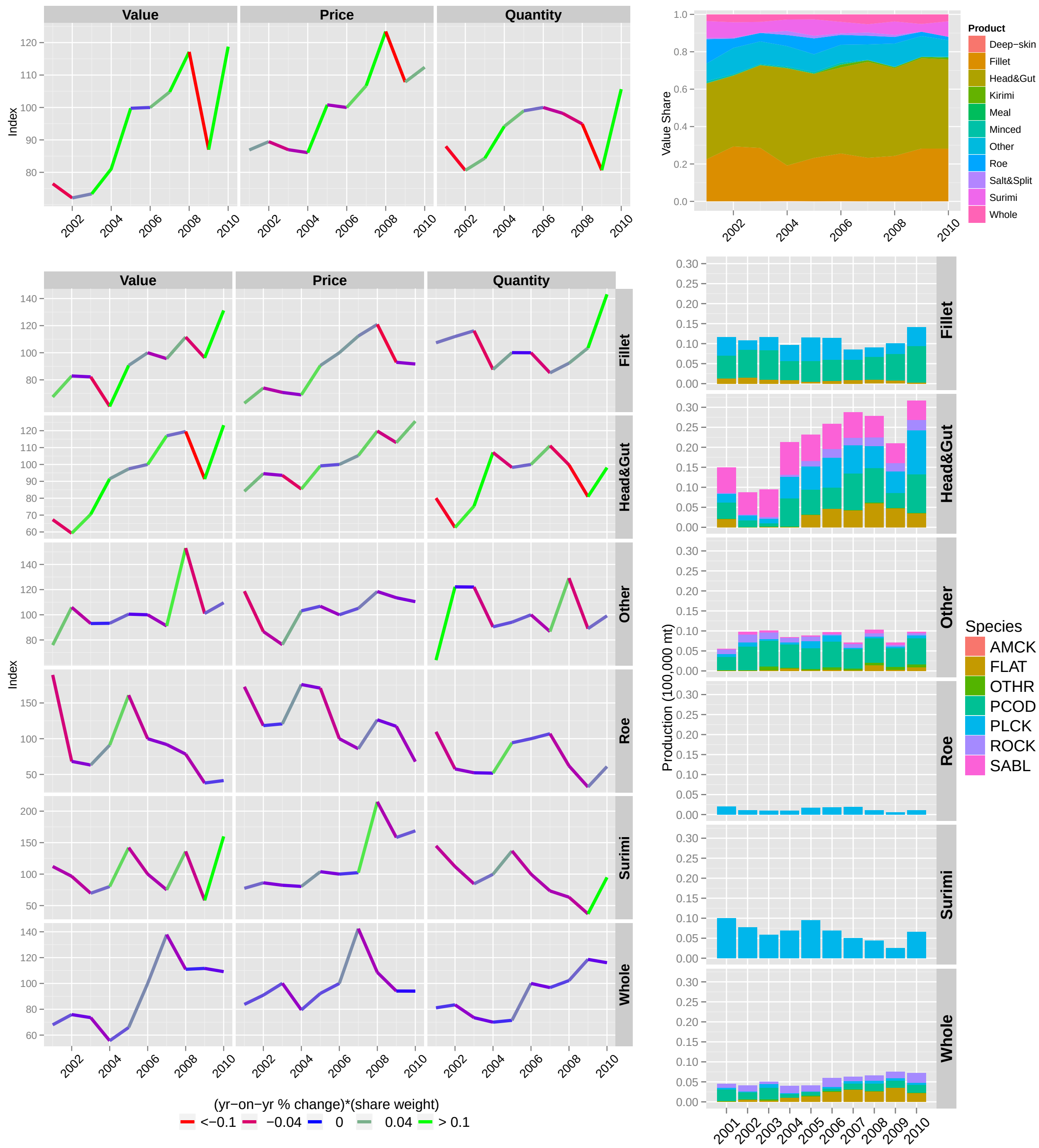

Figure 4. -- Gulf of Alaska shoreside wholesale market: product decomposition 2001−2010. upper-left panel: Aggregate value, price and quantity indices, base(2006)=100
lower-left panel: Product attribute indices (rows) of value, price and quantity indices (columns), base(2006)=100
upper-right panel: Annual contemporaneous value share by product
lower-right panel: Annual production (100,000 metric tons) of products by species type

Appendix

Attribute Decomposition Proofs

Proof of: Value-share decomposition of the Laspeyres Index

Let J and K be attributes as defined in the methods section and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Laspeyres Index $I^L(1|0)$ over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^L(1|0)$ over an attribute J or $I_k^L(1|0)$ over attribute K.

$$I^L(0|1) = \frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1), y_{j,k}(0))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0), y_{j,k}(0))} = \sum_{j\in J} I_j^L(0|1) * S_j^L = \sum_{k\in K} I_k^L(0|1) * S_k^L$$
$$\text{where} \quad I_j^L(1|0) = \frac{R_j(x_{j,k}(1), y_{j,k}(0))}{R_j(x_{j,k}(0), y_{j,k}(0))} \quad S_j^L = \frac{R_j(x_{j,k}(0), y_{j,k}(0))}{\sum_{j\in J} R_j(x_{j,k}(0), y_{j,k}(0))} \tag{18}$$
$$\text{and} \quad R_j(x_{j,k}(t), y_{j,k}(s)) = \sum_{k\in K} R_{j,k}(x_{j,k}(t), y_{j,k}(s))$$
$$\text{with} \quad I_k^L,\ S_k^L,\ R_k(x_{j,k}(t), y_{j,k}(s)) \quad \text{equivalently defined.}$$

Proof:

$$\begin{aligned}
I^L(0|1) &= \frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1), y_{j,k}(0))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0), y_{j,k}(0))} = \sum_{j\in J} R_j(x_{j,k}(1), y_{j,k}(0)) \frac{1}{\sum_{j\in J} R_j(x_{j,k}(0), y_{j,k}(0))} \\
&= \left(\sum_{j\in J} R_j(x_{j,k}(1), y_{j,k}(0))\right) * \frac{R_j(x_{j,k}(0), y_{j,k}(0))}{R_j(x_{j,k}(0), y_{j,k}(0))} * \frac{1}{\sum_{j\in J} R_j(x_{j,k}(0), y_{j,k}(0))} \\
&= \sum_{j\in J} \frac{R_j(x_{j,k}(1), y_{j,k}(0))}{R_j(x_{j,k}(0), y_{j,k}(0))} * \frac{R_j(x_{j,k}(0), y_{j,k}(0))}{\sum_{j\in J} R_j(x_{j,k}(0), y_{j,k}(0))} \\
&= \sum_{j\in J} I_j^L(1|0) * S_j^L
\end{aligned} \tag{19}$$

The decomposition across the K attribute dimension, $I^L(0|1) = \sum_{j\in J} I_k^L(1|0) * S_k^L$, is accomplished by carrying out the same set of operations over k instead. □

Proof of: Value-share decomposition of the Paasche Index

Let J and K be attributes as defined in the methods section and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Paasche Index $I^A(1|0)$

over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^A(1|0)$ over an attribute J or $I_k^A(1|0)$ over attribute K.

$$\begin{aligned}
&I^A(0|1) = \frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1),y_{j,k}(1))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0),y_{j,k}(1))} = \sum_{j\in J} I_j^A(0|1) * S_j^A = \sum_{k\in K} I_k^A(0|1) * S_k^A \\
&\text{where} \quad I_j^A(1|0) = \frac{R_j(x_{j,k}(1),y_{j,k}(1))}{R_j(x_{j,k}(0),y_{j,k}(1))} \quad S_j^A = \frac{R_j(x_{j,k}(0),y_{j,k}(1))}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(1))} \\
&\text{and} \quad R_j(x_{j,k}(t),y_{j,k}(s)) = \sum_{k\in K} R_{j,k}(x_{j,k}(t),y_{j,k}(s)) \\
&\text{with} \quad I_k^A,\ S_k^A,\ R_k(x_{j,k}(t),y_{j,k}(s)) \quad \text{equivalently defined.}
\end{aligned} \tag{20}$$

Proof:

$$\begin{aligned}
I^A(0|1) &= \frac{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(1),y_{j,k}(1))}{\sum_{j\in J}\sum_{k\in K} R_{j,k}(x_{j,k}(0),y_{j,k}(1))} = \sum_{j\in J} R_j(x_{j,k}(1),y_{j,k}(1)) \frac{1}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(1))} \\
&= \left(\sum_{j\in J} R_j(x_{j,k}(1),y_{j,k}(1))\right) * \frac{R_j(x_{j,k}(0),y_{j,k}(1))}{R_j(x_{j,k}(0),y_{j,k}(1))} * \frac{1}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(1))} \\
&= \sum_{j\in J} \frac{R_j(x_{j,k}(1),y_{j,k}(1))}{R_j(x_{j,k}(0),y_{j,k}(1))} * \frac{R_j(x_{j,k}(0),y_{j,k}(1))}{\sum_{j\in J} R_j(x_{j,k}(0),y_{j,k}(1))} \\
&= \sum_{j\in J} I_j^A(1|0) * S_j^A
\end{aligned} \tag{21}$$

The decomposition across the K attribute dimension, $I^A(0|1) = \sum_{j\in J} I_k^A(1|0) * S_k^A$, is accomplished by carrying out the same set of operations over k instead. □

Proof of: Value-share decomposition of the Fisher Index

Let J and K be attributes as defined in the methods section and $(x_{j,k}(t), y_{j,k}(t))$ a set of double array variables for $t = 0, 1$. The aggregate Fisher Index $I^F(1|0)$ over variable $x_{j,k}(t)$ using $y_{j,k}(t)$ to construct weights can be linearly decomposed into sub-indices $I_j^F(1|0)$ over an attribute J or $I_k^F(1|0)$ over attribute K.

$$\begin{aligned}
&I^F(0|1) = \sqrt{I^A(1|0) * I^L(1|0)} = \sum_{j\in J} I_j^F(0|1) * S_j^F = \sum_{k\in K} I_k^F(0|1) * S_k^F \\
&\text{where} \quad I_j^F(1|0) = \sqrt{I_j^A(1|0) * I_j^L(1|0)} \quad S_j^F = \frac{S_j^A + S_j^L}{2} \\
&\text{with } I_j^L(1|0), S_j^L \text{ defined as in Equation (1) and} \\
&I_j^A(1|0), S_j^A \text{ defined as in Equation (2)} \\
&\text{The } K \text{ attribute decomposition using } I_k^F,\ S_k^F \text{ is equivalently defined.}
\end{aligned} \tag{22}$$

Proof:

The strategy for showing the Fisher decomposition will utilize the Laspeyres and Paasche indices which have straightforward value-share decompositions. The linear approximation the index $I(x(1), x(0), y(1), y(0))$ is taken with respect to the current period argument $x(1)$ about it's base value $x(0)$, and can be thought of as the temporal increment of x from $t = 0$. The approximation conditions on the variables $(x(0), y(1), y(0))$. To emphasize this, and that x a variable (rather than a observation), the following notation $I(x|x(0), y(1), y(0)) = I(x)$ is adopted.

To make the approximation slightly easier define $h(x) \equiv I^F(x)^2 = I^L(x) * I^A(x)$.

The derivative of $h(x)$ w.r.t x is

$$\frac{dh(x)}{dx} = \frac{dI^L}{dx}I^A + \frac{dI^A}{dx}I^L, \tag{23}$$

by the chain rule

$$\frac{dh(x)}{dx} = \frac{dh}{dI^F}\frac{dI^F}{dx} = 2\,\frac{dI^F}{dx} \quad \text{thus} \quad \left(\frac{1}{2}\right)\frac{dh(x)}{dx} = \frac{dI^F}{dx}. \tag{24}$$

Using Equation (23) in Equation (24) gives

$$\frac{dI^F}{dx} = \frac{1}{2}\left(\frac{dI^L}{dx}I^A + \frac{dI^A}{dx}I^L\right).$$

A first-order Taylor series approximation to the Fisher Index is

$$I^F(x) \approx I^F|_{x=x(0)} + \frac{1}{2}\left(\frac{dI^L}{dx}I^A + \frac{dI^A}{dx}I^L\right)\Bigg|_{x=x(0)}(x - x(0)). \tag{25}$$

Note that if there is no change in the value x from $x(0)$, (i.e., x = $x(0)$) then $I^L = I^A = I^F = 1$ (i.e., no change).[16] Thus,

$$I^F|_{x=x(0)} = 1 \quad \text{and} \quad \left(\frac{dI^L}{dx}I^A + \frac{dI^A}{dx}I^L\right)\Bigg|_{x=x(0)} = \frac{dI^L}{dx}\Bigg|_{x=x(0)} + \frac{dI^A}{dx}\Bigg|_{x=x(0)}.$$

[16]Similarly since the Laspeyres and Paasche indices only differ in their use of the weighting variable, if $y(1) = y(0)$ then $I^L = I^A = I^F$, however this is not necessarily equal to 1.

Using this result in Equation (25) is

$$I^F(x) \approx 1 + \frac{1}{2}\left(\left.\frac{dI^L}{dx}\right|_{x=x(0)} + \left.\frac{dI^A}{dx}\right|_{x=x(0)} \right)(x - x(0))\,. \tag{26}$$

The derivative of the Laspeyres and Paasche indices are

$$\frac{dI^L}{dx} = \frac{y(0)}{x(0)y(0)} \quad \frac{dI^A}{dx} = \frac{y(1)}{x(0)y(1)},$$

thus, the right hand side of Equation (26) is[17]

$$\begin{aligned} 1 + \frac{1}{2}\left(\frac{y(0)}{x(0)y(0)} + \frac{y(1)}{x(0)y(1)}\right)(x - x(0)) &= 1 + \frac{1}{2}\left(\frac{x(0)y(0)}{x(0)y(0)} + \frac{x(0)y(1)}{x(0)y(1)}\right)\left(\frac{x - x(0)}{x(0)}\right) \\ &= 1 + \left(\frac{x - x(0)}{x(0)}\right). \end{aligned} \tag{27}$$

Therefore, the first-order approximation of the Fisher Index is one plus the percentage change in the index variable x from it's base value $x(0)$:

$$I^F(x) \approx 1 + \frac{x - x(0)}{x(0)}. \tag{28}$$

The first-order approximation can be used to extend the attribute decomposition to the Fisher Index. Starting from the attribute J decomposition of the Laspeyres and Paasche indices (Equations (19) and (21) respectively) we can write the Fisher Index as

$$I^F = \sqrt{I^L * I^A} = \sqrt{\left(\sum_{j\in J} I_j^L S_j^L\right) * \left(\sum_{j\in J} I_j^A S_j^A\right)}. \tag{29}$$

The derivative of $h(x)$ w.r.t x now becomes

$$\frac{dh(x)}{dx} = \sum_{j\in J} \frac{\partial h(x_j)}{\partial x_j} = \sum_{j\in J} \frac{\partial I_j^L}{\partial x_j} S_j^L I_j^A + \frac{\partial I_j^A}{\partial x_j} S_j^L I_j^L. \tag{30}$$

[17]Note that $\frac{dI^L}{dx}$ and $\frac{dI^L}{dx}$ do not depend on x, so $\frac{dI^L}{dx}|_{x=x(0)} = \frac{dI^L}{dx}$ and $\frac{dI^A}{dx}|_{x=x(0)} = \frac{dI^A}{dx}$.

So that the first-order approximation is

$$\begin{aligned}
I^F(x) &\approx I^F|_{x=x(0)} + \sum_{j\in J} \frac{1}{2}\left(\frac{\partial I_j^L}{\partial x_j} I_j^A + \frac{\partial I_j^A}{\partial x_j} I_j^L\right)\Big|_{x_j=x_j(0)} (x_j - x_j(0)) \\
&\approx 1 + \sum_{j\in J} \frac{1}{2}\left(\frac{y_j(0)}{x_j(0)y_j(0)} S_j^L + \frac{y_j(1)}{x_j(0)y_j(1)} S_j^A\right)(x_j - x_j(0)) \\
&\approx 1 + \sum_{j\in J} \frac{1}{2}\left(S_j^L + S_j^A\right) \frac{x_j - x_j(0)}{x_j(0)}. \qquad (31)
\end{aligned}$$

The attribute decomposition of the Fisher Index shows that it is a value-share weighted average of the percentage changes in each of the index variables where the share weight is the average of the Laspeyres and Paasche value shares. We can make the Fisher attribute decomposition look more like the Laspeyres and Paasche decompositions by defining $S_j^F \equiv \frac{1}{2}\left(S_j^L + S_j^A\right)$:

$$\begin{aligned}
I^F &\approx 1 + \sum_{j\in J} \frac{1}{2}\left(S_j^L + S_j^A\right) \frac{x_j - x_j(0)}{x_j(0)} \\
&\approx \sum_{j\in J} S_j^F + \sum_{j\in J} S_j^F \frac{x_j - x_j(0)}{x_j(0)} \\
&\approx \sum_{j\in J} S_j^F \left(1 + \frac{x_j - x_j(0)}{x_j(0)}\right) \\
&\approx \sum_{j\in J} S_j^F I_j^F \qquad \text{where} \quad I_j^F \equiv \sqrt{I_j^L * I_j^A}. \qquad (32)
\end{aligned}$$

The decomposition across the K attribute dimension, $I^F(0|1) = \sum_{j\in J} I_k^F(1|0) * S_k^F$, is accomplished by starting with the Laspeyres and Paasche decompositions across attribute K in Equation (29) and carrying out the same set of operations over k instead. $\square$

Proof of: Decomposition of the chained index

Let J be an attribute as defined in the methods section with classes $j \in J$ and $(x_j(t), y_j(t))$ a set of double array variables for $t = 0, 1, \ldots, T$. The aggregate chained index, $\mathbf{I}(0, \tilde{T})$, (Equation (11)) can be approximately decomposed by class j as product of the value-share weighted geometric mean over time of the sub-index increments $I_j(t|t–1)$:

$$\mathbf{I}(T,0) \approx \prod_{j\in J}\prod_{t=1}^{T} I_j(t|t\text{–}1)^{S_j(t|t-1)}. \quad (33)$$

Proof:

This derivation shows the relationship between the aggregate chained index and underlying chained sub-indices of an attribute. The derivation uses the following common first-order approximation of the logarithm: let $x = 1 + \Delta$ with Δ small, then

$\log(x) = \log(1 + \Delta) \approx \Delta = x - 1$:

$$\log(\mathbf{I}(T,0)) = \sum_{t=1}^{T} \log(I(t|t\text{–}1)) \approx \sum_{t=1}^{T} I(t|t\text{–}1) - 1.$$

Using the linear decomposition of the Laspeyres, Paasche or Fisher Indexes (presented in the section Relating Disaggregate and Aggregate Economic Indices) and a convenient way of writing the number one, $1 = \sum_j S_j(t|t\text{–}1)$:

$$\begin{aligned}
\log(\mathbf{I}(T,0)) &\approx \sum_{t=1}^{T}\sum\nolimits_j I_j(t|t\text{–}1)S_j(t|t\text{–}1) - \sum\nolimits_j S_j(t|t\text{–}1) \\
&= \sum_{t=1}^{T}\sum\nolimits_j (I_j(t|t\text{–}1) - 1)S_j(t|t\text{–}1) \\
&\approx \sum_{t=1}^{T}\sum\nolimits_j \log(I_j(t|t\text{–}1))\, S_j(t|t\text{–}1).
\end{aligned}$$

Taking the exponential shows that the aggregate chain index can be approximately decomposed into value-share weighted geometric means over time for each class j in attribute J, $\prod_{t=1}^{T} I_j(t|t\text{–}1)^{S_j(t|t\text{-}1)}$:

$$\mathbf{I}(T,0) \approx \prod_{j\in J}\prod_{t=1}^{T} I_j(t|t\text{–}1)^{S_j(t|t\text{-}1)}. \quad (34)$$

□

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